Model Predictive Control of Water Networks

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HYCON/EFFINET School
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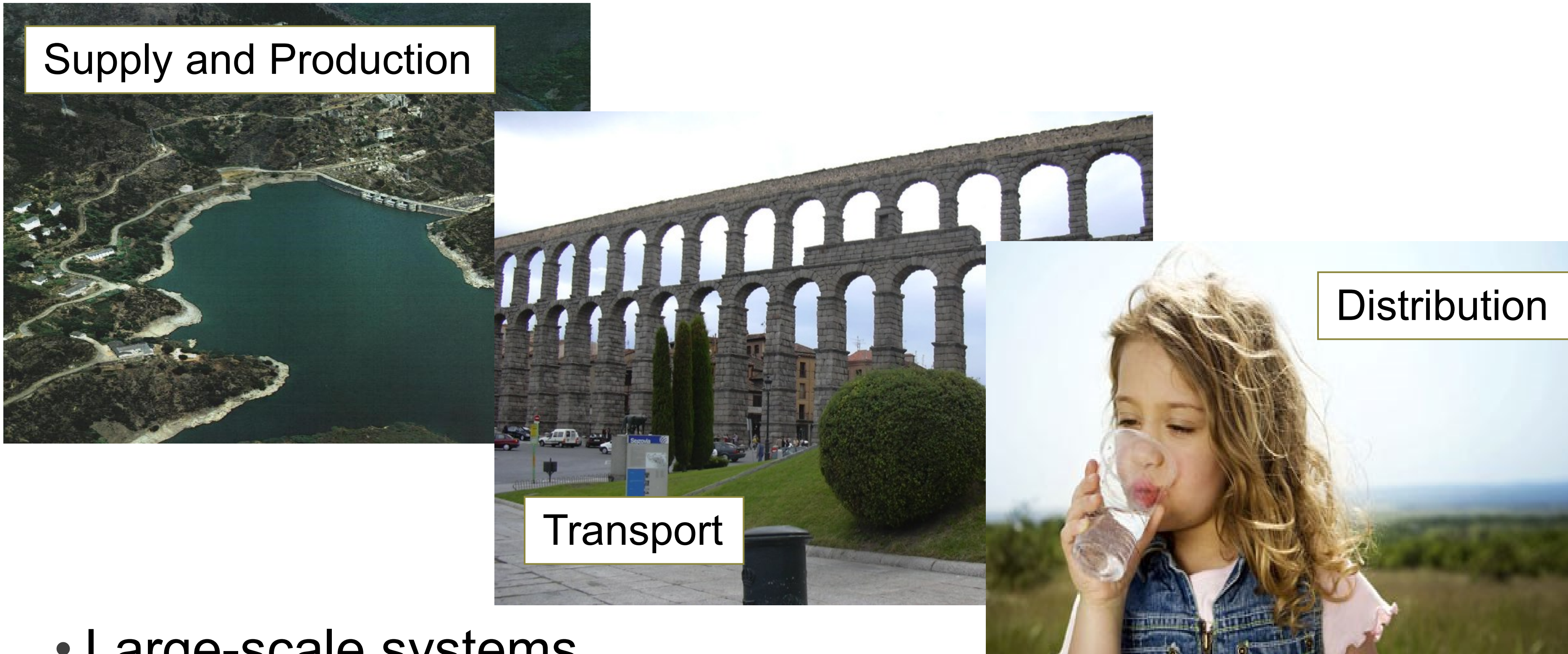
SEVENTH FRAMEWORK PROGRAMME

Introduction

Ocampo-Martinez, C.; Puig, V.; Cembrano, M.; Quevedo, J. "Application of predictive control strategies to the management of complex networks in the urban water cycle". IEEE Control Systems Magazine.33 - 1, pp. 15 -41. 2013. I SSN 1066-033X.

The Water Cycle

Drinking Water Networks



- Large-scale systems
- Complex dynamic models (non-linear, hybrid)
- Management and control techniques: centralized scheme
- Complex controllers, even un-scalable (due to their system model)

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M. Brdys and B. Ulanicki, *Operational Control of Water Systems: Structures, algorithms and applications. UK: Prentice Hall International,* 1994

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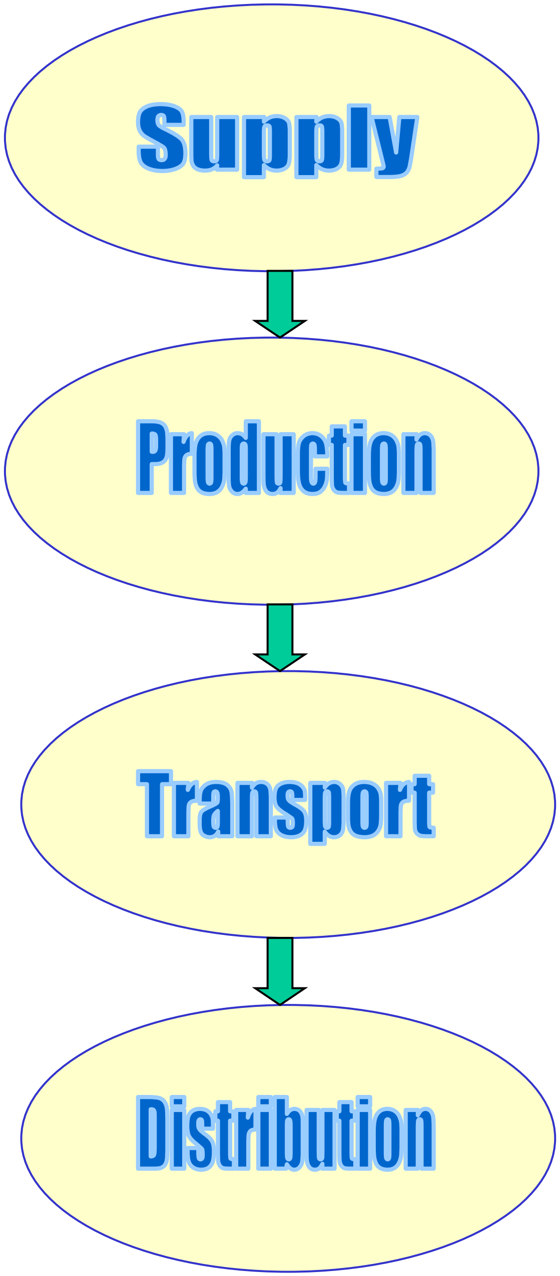
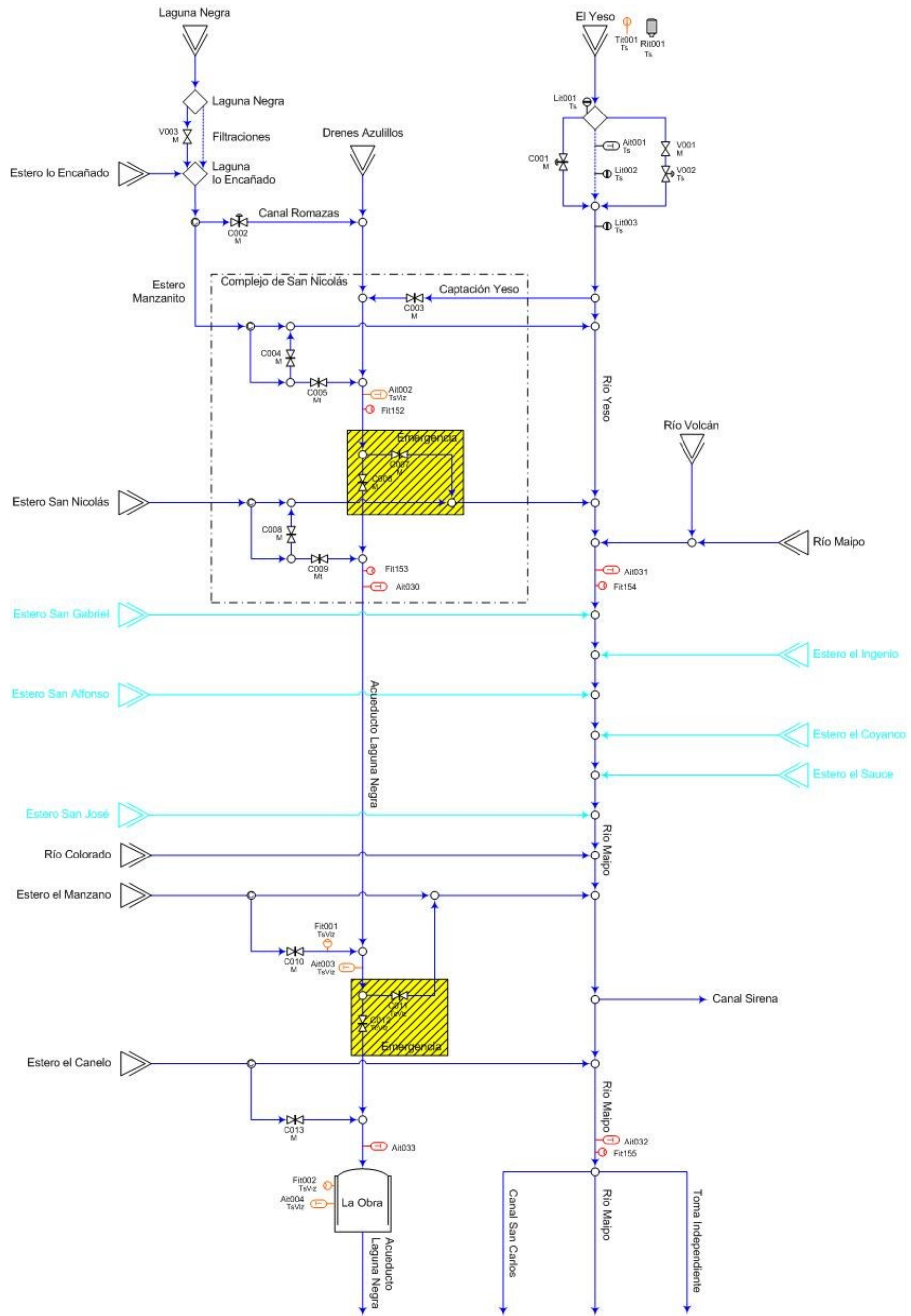
Hierarchy of Water Networks



Supply Network



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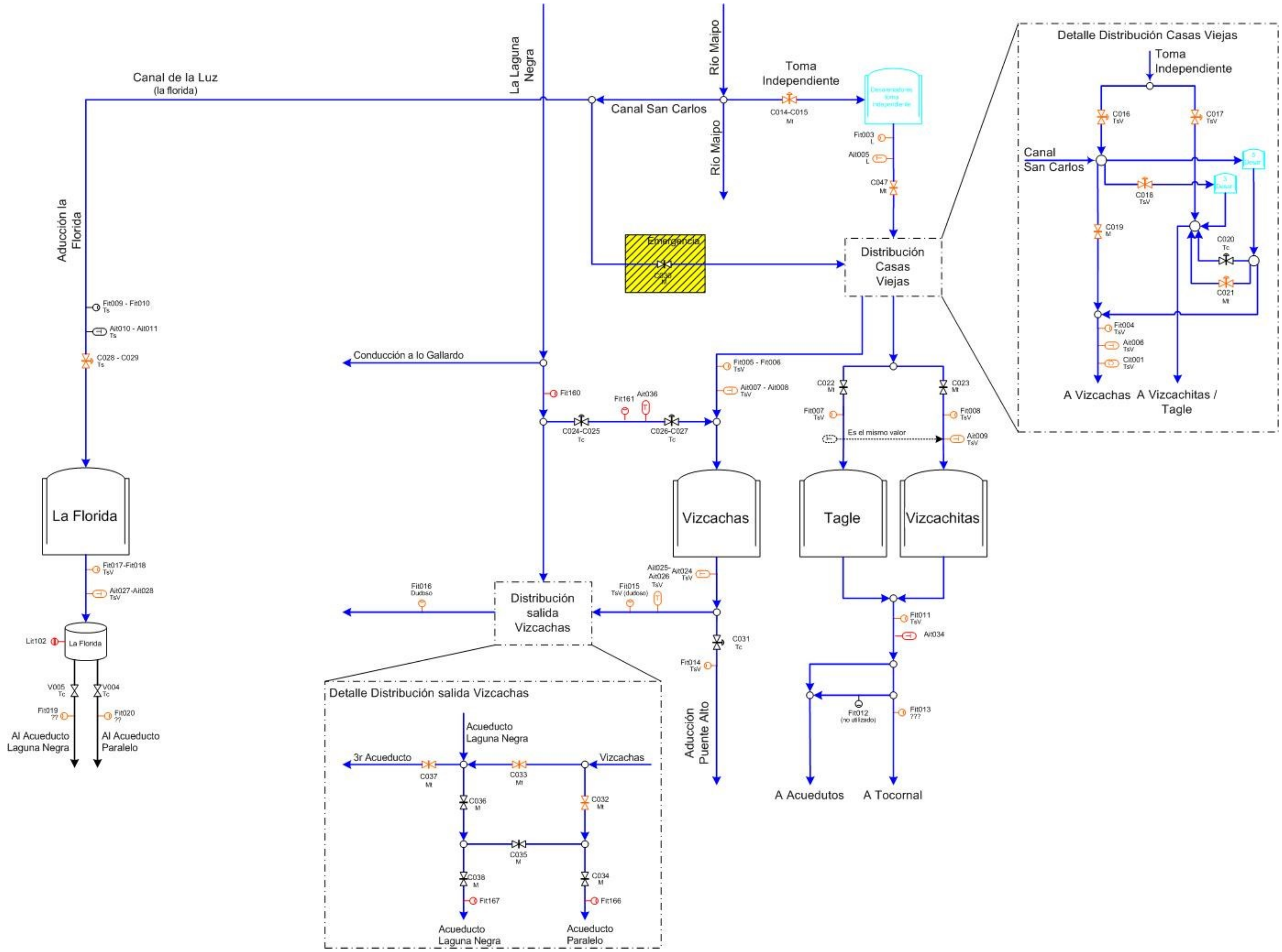
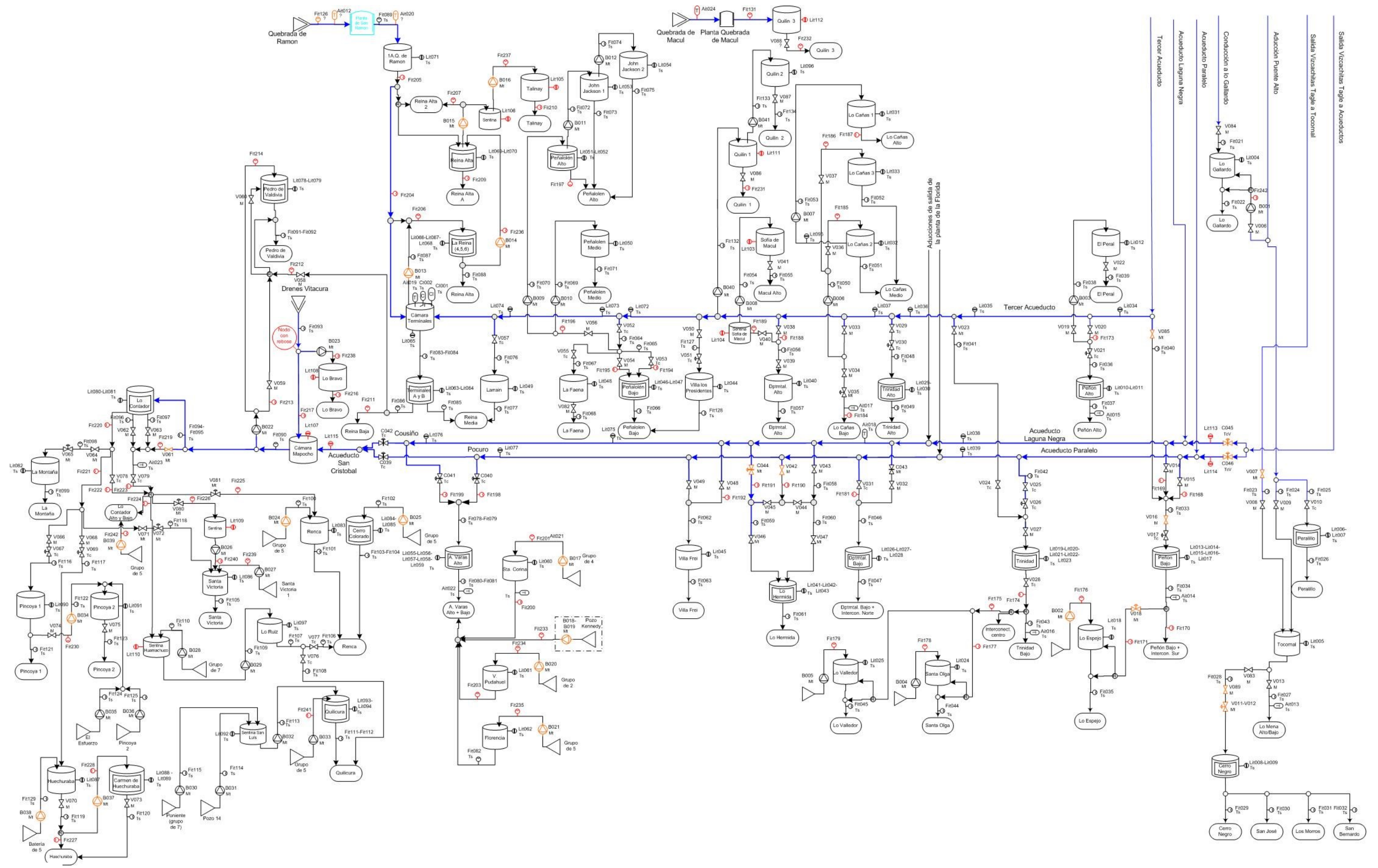
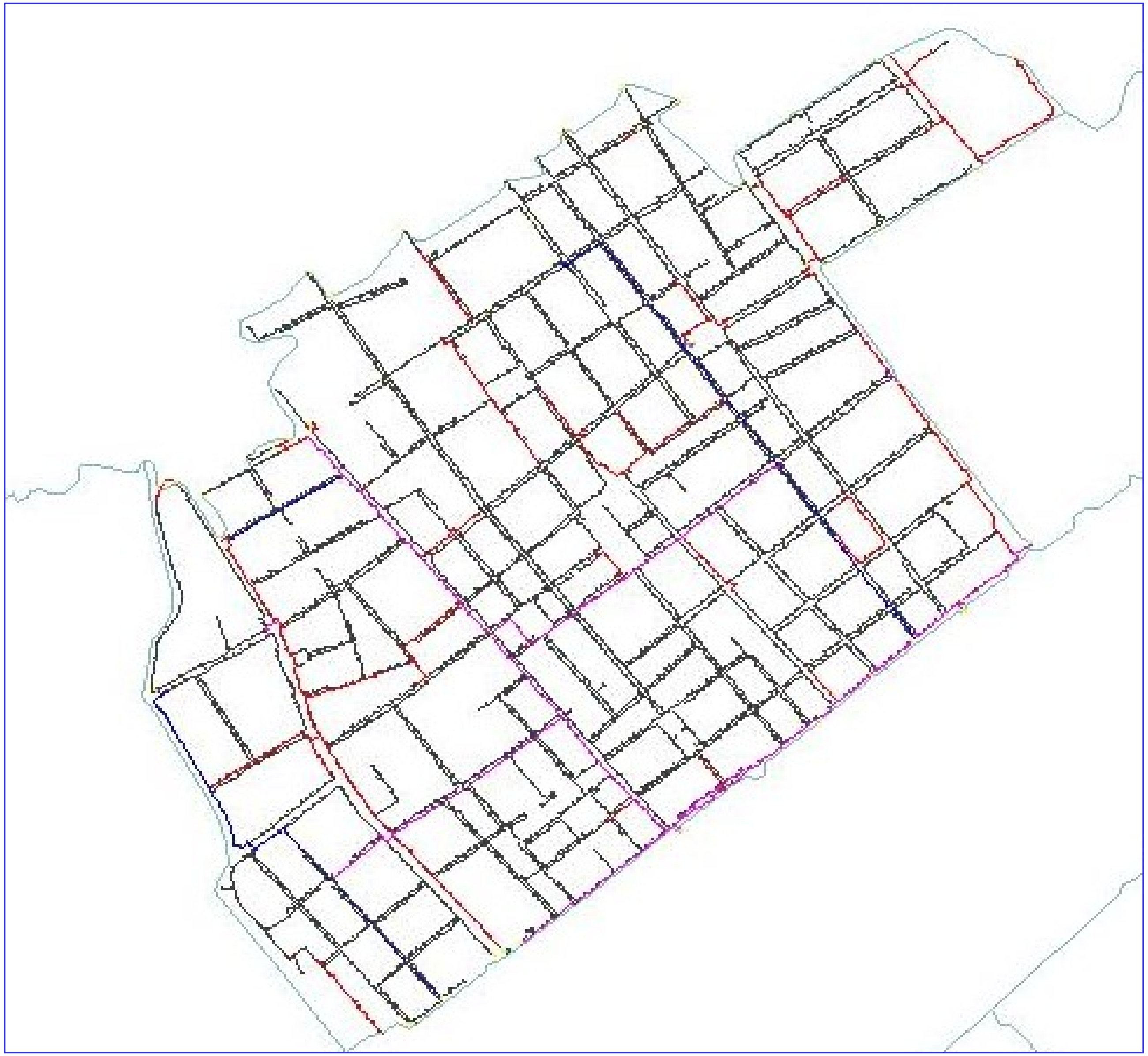
Production Network



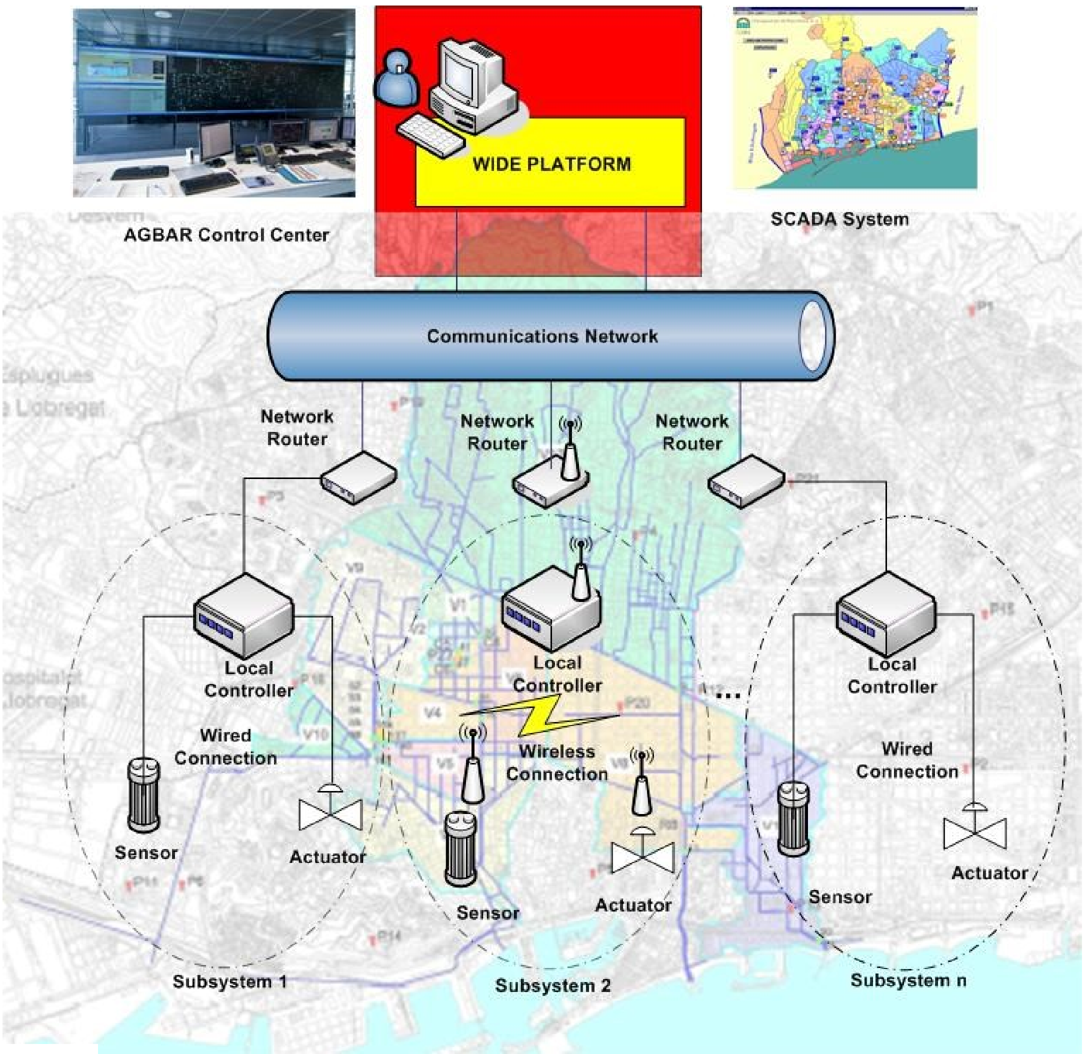
Transport Network

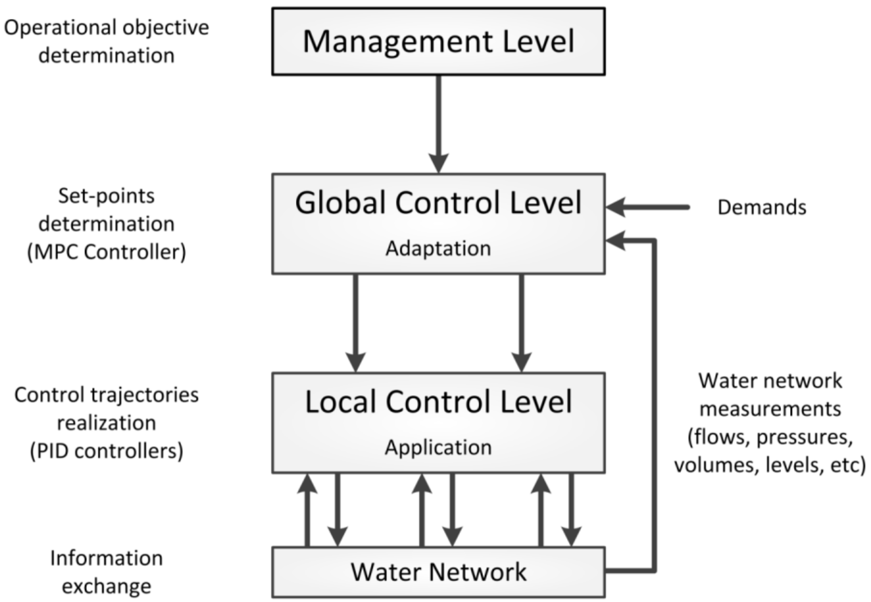


Distribution Network



The Role of MPC in Water Networks: Supervisory Control





MPC of Water Transport Networks: The Barcelona Case Study

J. Pascual, , J. Romera, , V. Puig, G. Cembrano, Operational predictive optimal control of Barcelona water transport network. Control Engineering Practice Volume 21, Issue 8, August 2013, Pages 1020–1034

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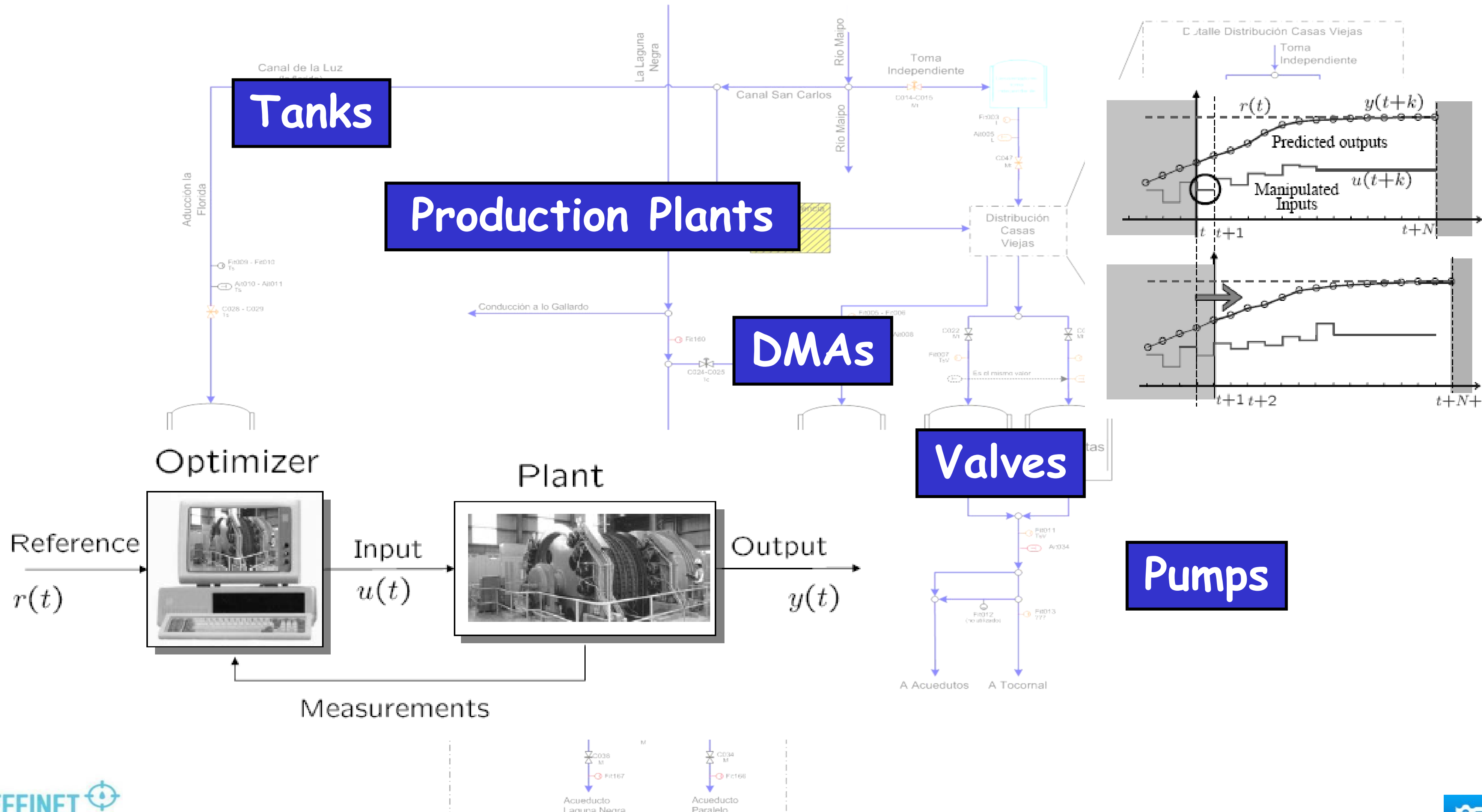
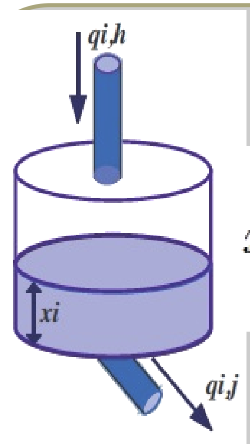
Elements of a Water Transport Network



Control Oriented Modelling: Flow-based model

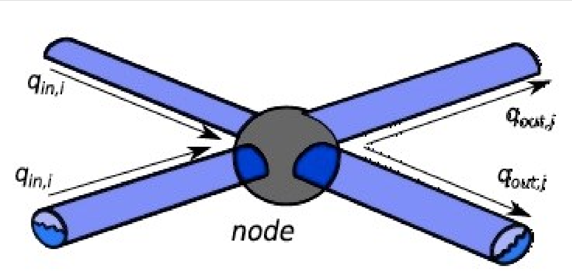
Reservoirs



$$x_i(k+1) = x_i(k) + \Delta t \left(\sum_i q_{\text{in},i}(k) - \sum_j q_{\text{out},j}(k) \right)$$

$$x_i^{\min} \leq x_i(k) \leq x_i^{\max}$$

Network Nodes



$$\sum_i q_{\text{in},i}(k) = \sum_j q_{\text{out},j}(k)$$

Network Actuators

$$u^{min} \leqslant u(k) \leqslant u^{max}$$

n states *x* (volumes)
m inputs *u* (actuator flows)
p disturbances (water demands)

Flow-based Linear Model

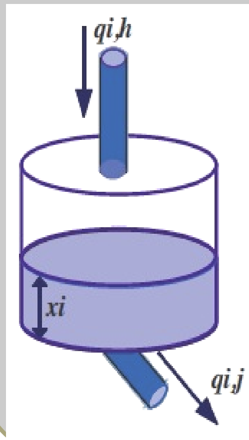
$$\begin{aligned} x(k+1) &= Ax(k) + \Gamma v(k) \\ E_1 v(k) &= E_2 \end{aligned}$$

with

$$\Gamma = [B \quad B_p]$$
$$v(k) = [u(k)' \quad d(k)']'$$

Control Oriented Modelling: Pressure-based model

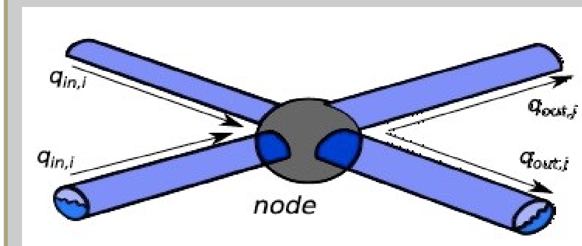
Reservoirs



$$\frac{\mathrm{d}V_i(t)}{\mathrm{d}t} = \sum_{p=1}^{M_i^{in}} q_{i,p}^{in}(t) - \sum_{l=1}^{M_i^{out}} q_{i,l}^{out}(t).$$

$$h_i(t) = \frac{V_i(t)}{S_i} + E_i$$

Pipes



$$q_{i,j} = u_{i,j} g_{i,j} (h_i - h_j) |h_i - h_j|^{-0.46}$$

Pumps

$$\Delta h = h_d - h_s = \begin{cases} Aq^2 + Bq + Cs^2 & if\ u \neq 0\ and\ s \neq 0 \\ 0 & otherwise \end{cases}$$

Valves

$$q_{i,j} = u_{i,j} g_{i,j} (h_i - h_j) |h_i - h_j|^{-0.46}$$

MPC Problem

Objective Function Formulation...

1. Energy/Production Costs

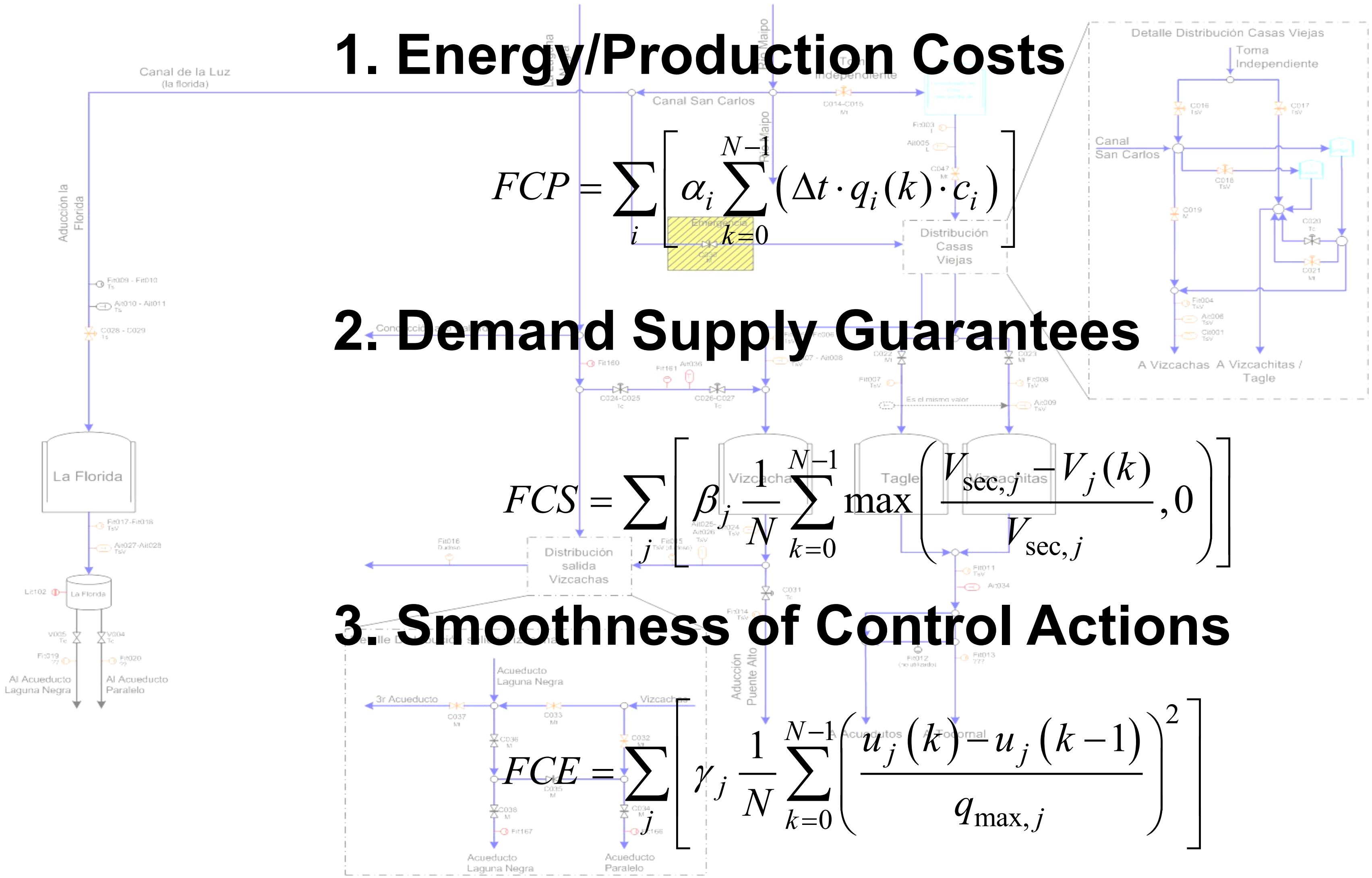
$$FCP = \sum_i \left[\alpha_i \sum_{k=0}^{N-1} \left(\Delta t \cdot q_i(k) \cdot c_i \right) \right]$$

2. Demand Supply Guarantees

$$FCS = \sum_j \left[\beta_j \frac{1}{N} \sum_{k=0}^{N-1} \max \left(\frac{V_{\sec,j} - V_j(k)}{V_{\sec,j}}, 0 \right) \right]$$

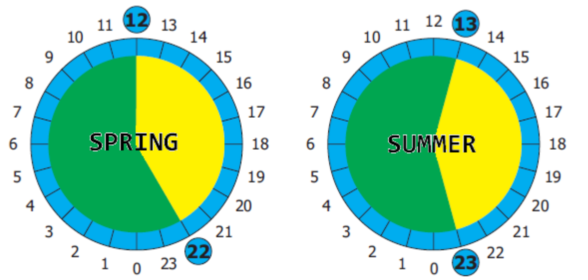
3. Smoothness of Control Actions

$$FCE = \sum_j \left[\gamma_j \frac{1}{N} \sum_{k=0}^{N-1} \left(\frac{u_j(k) - u_j(k-1)}{q_{\max,j}} \right)^2 \right]$$

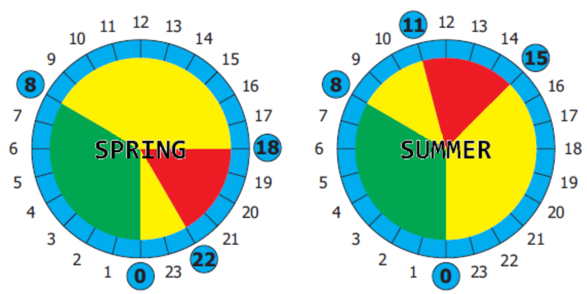


Electricity Cost Model

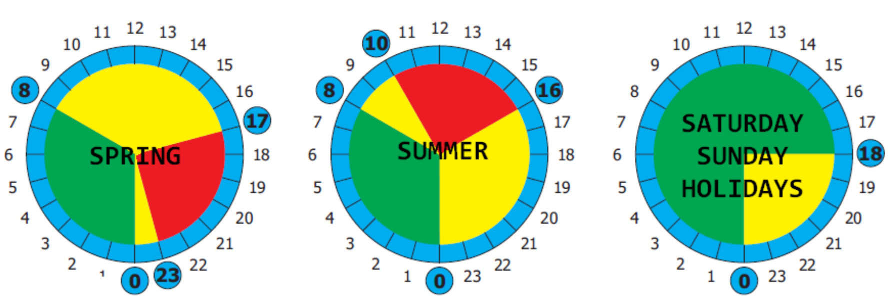
TIME TABLE: 1.0, 2.0.1, 2.0.2, 2.0.3, 3.0.1



TIME TABLE: 3.0.2

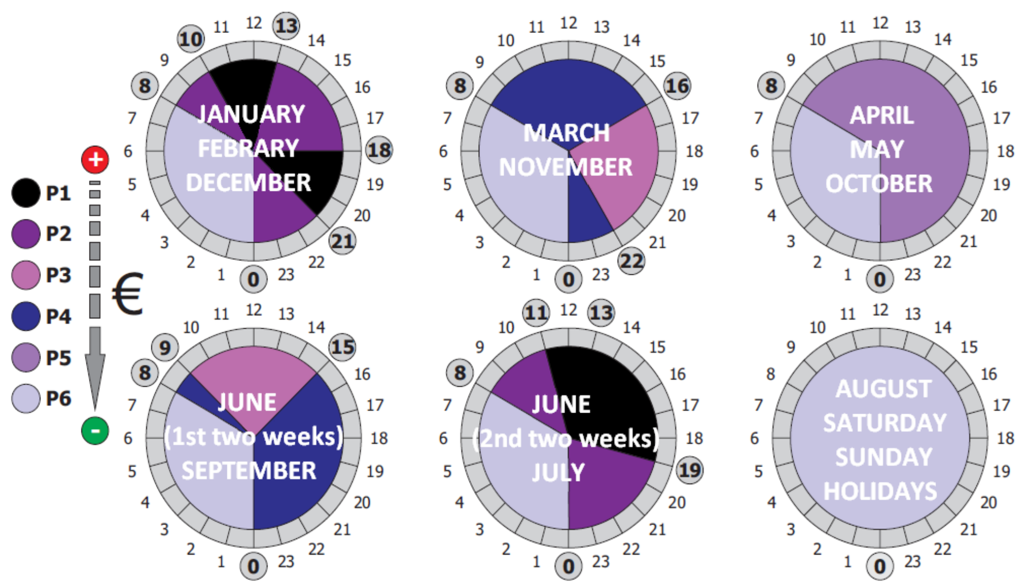


TIME TABLE: 3.1 < 450 KWh



- An electricity cost model that takes into account the price of electricity depending on the day, hour and period of the year has been developed and taken into account in the MPC formulation.

METHOD 6 PERIODS



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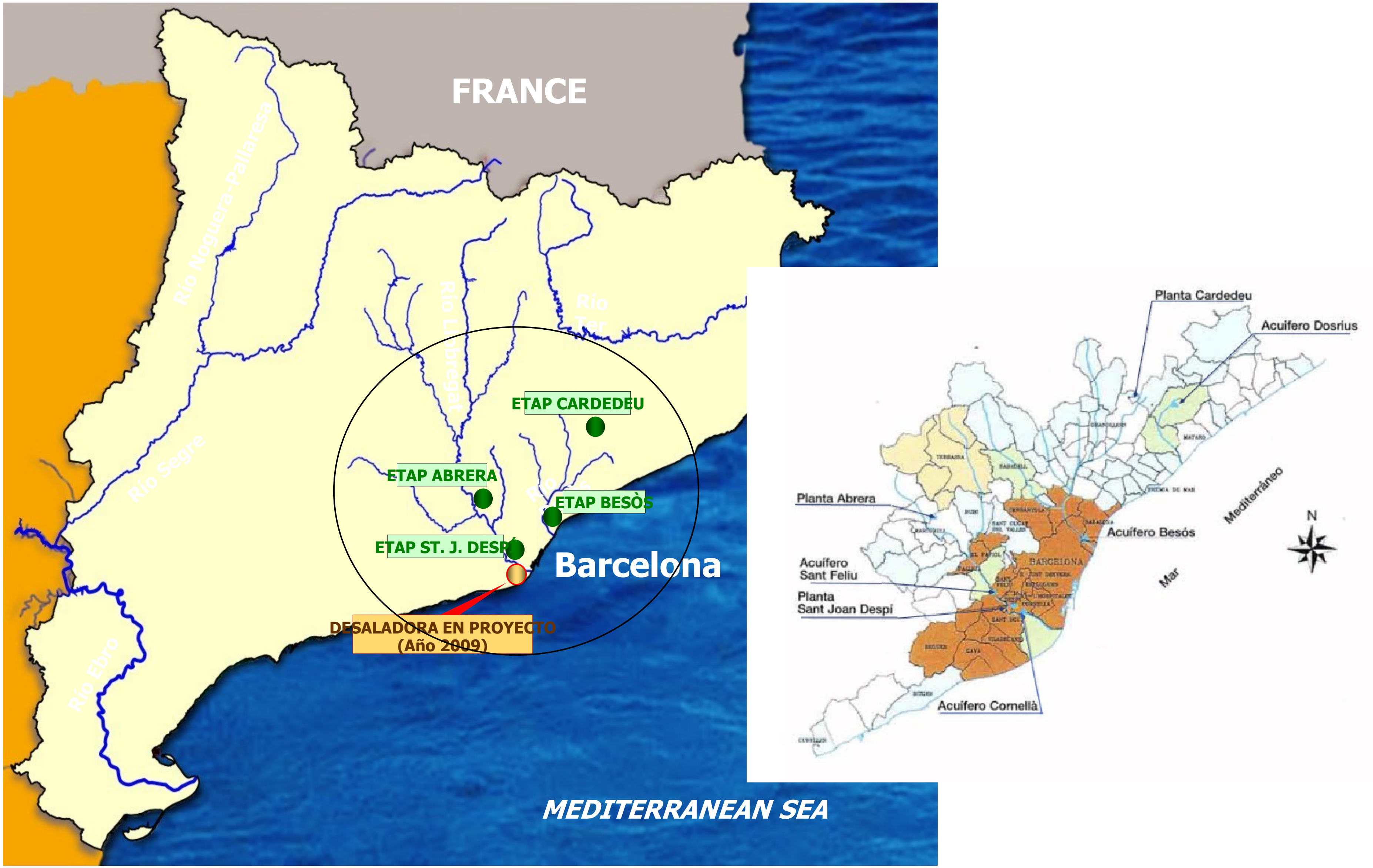
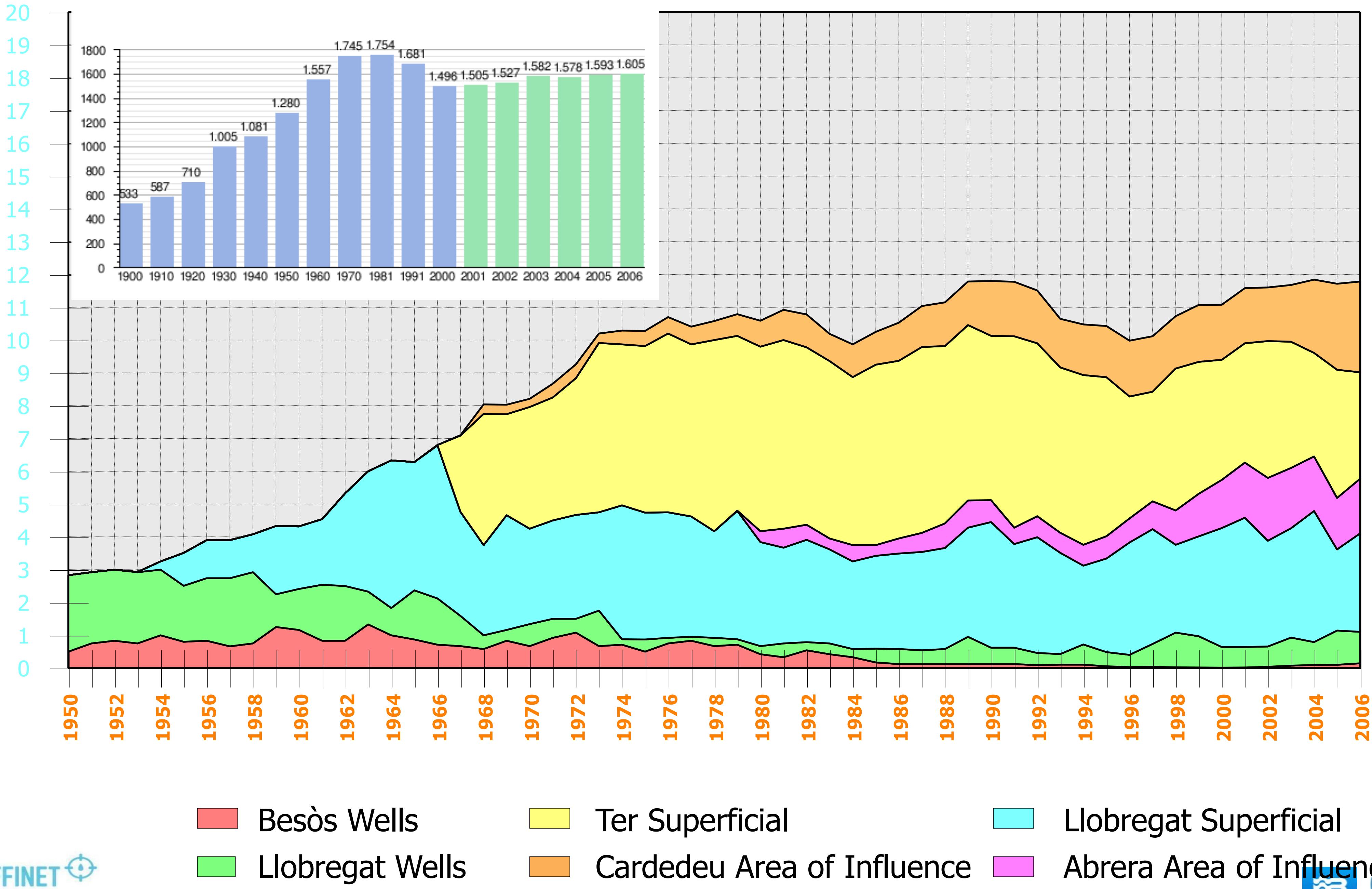
The Barcelona Case Study

Production Plants in Barcelona Network



Demand and Source Evolution



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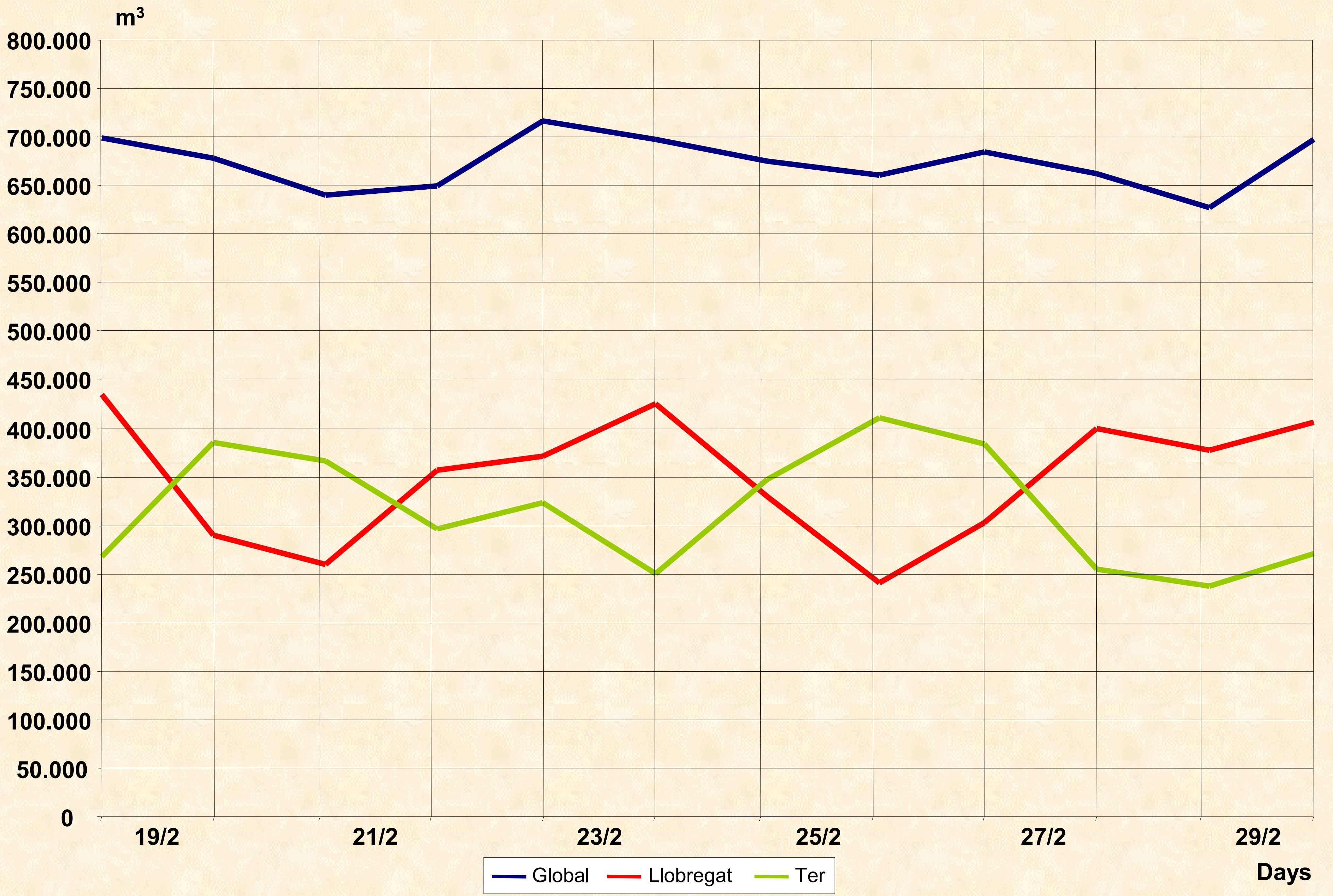
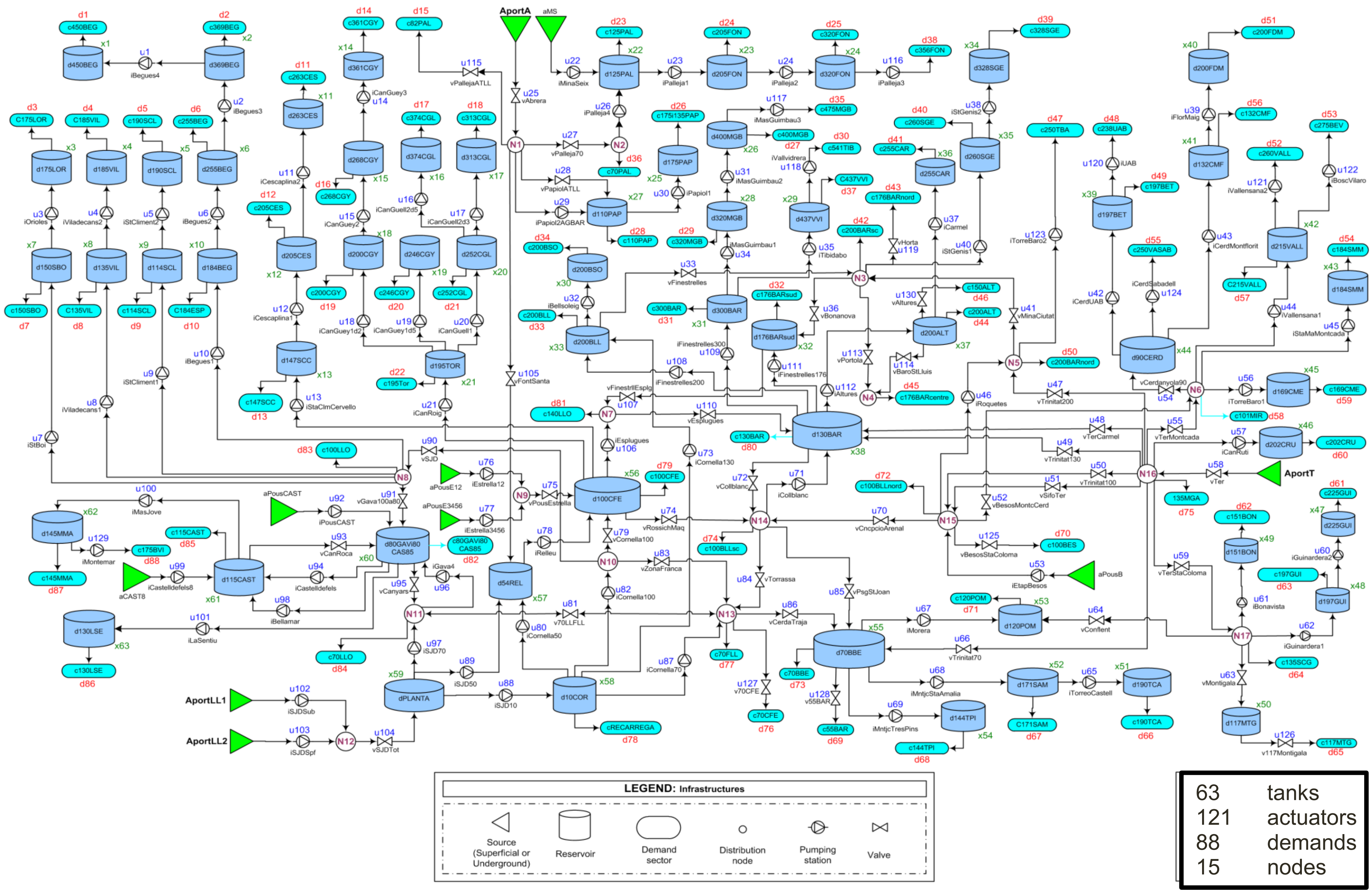
Demand by Source



Pressure Floors: DMAs

Barcelona Water Trasnport Network



63	tanks
121	actuators
88	demands
15	nodes

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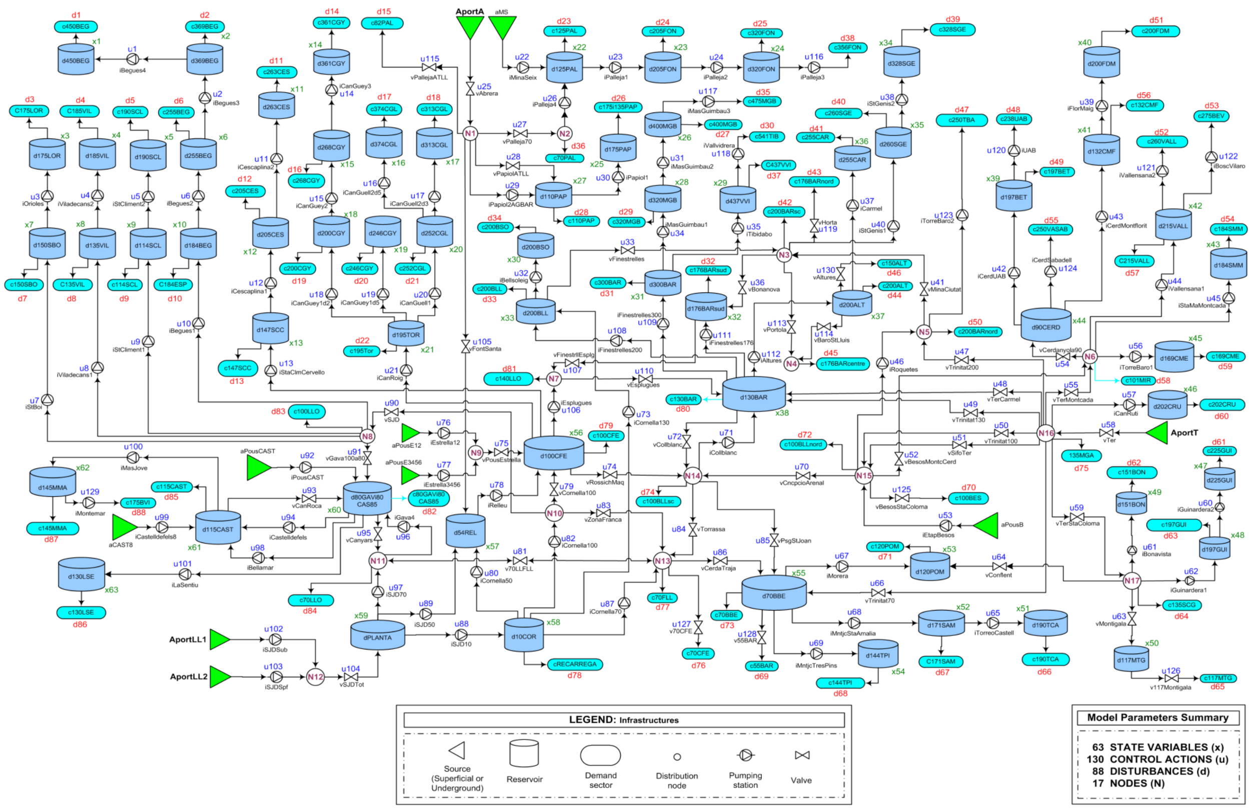
Some Numbers of Barcelona Water Transport Network



- General overview:

Municipalities supplied	23
Supply area	424 km^2
Population supplied	2.922.773
Average demand	7 m^3/s

- Network parameters:

Pipes length	4.645 km
Pressure floors	113
Sectors	218

- Facilities

Remote stations	98
Water storage tanks	81
Valves	64
Flow meters	92
Pumps / Pumping stations	180 / 84
Chlorine dosing devices	23
Chlorine analyzers	74

Different sources with different prices
- Superfitial sources → **X** €/m^3
- Underground sources → **0,6 X** €/m^3
- Desalination → **4,0 X** €/m^3

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Demand Forecast for MPC of Water Transport Networks

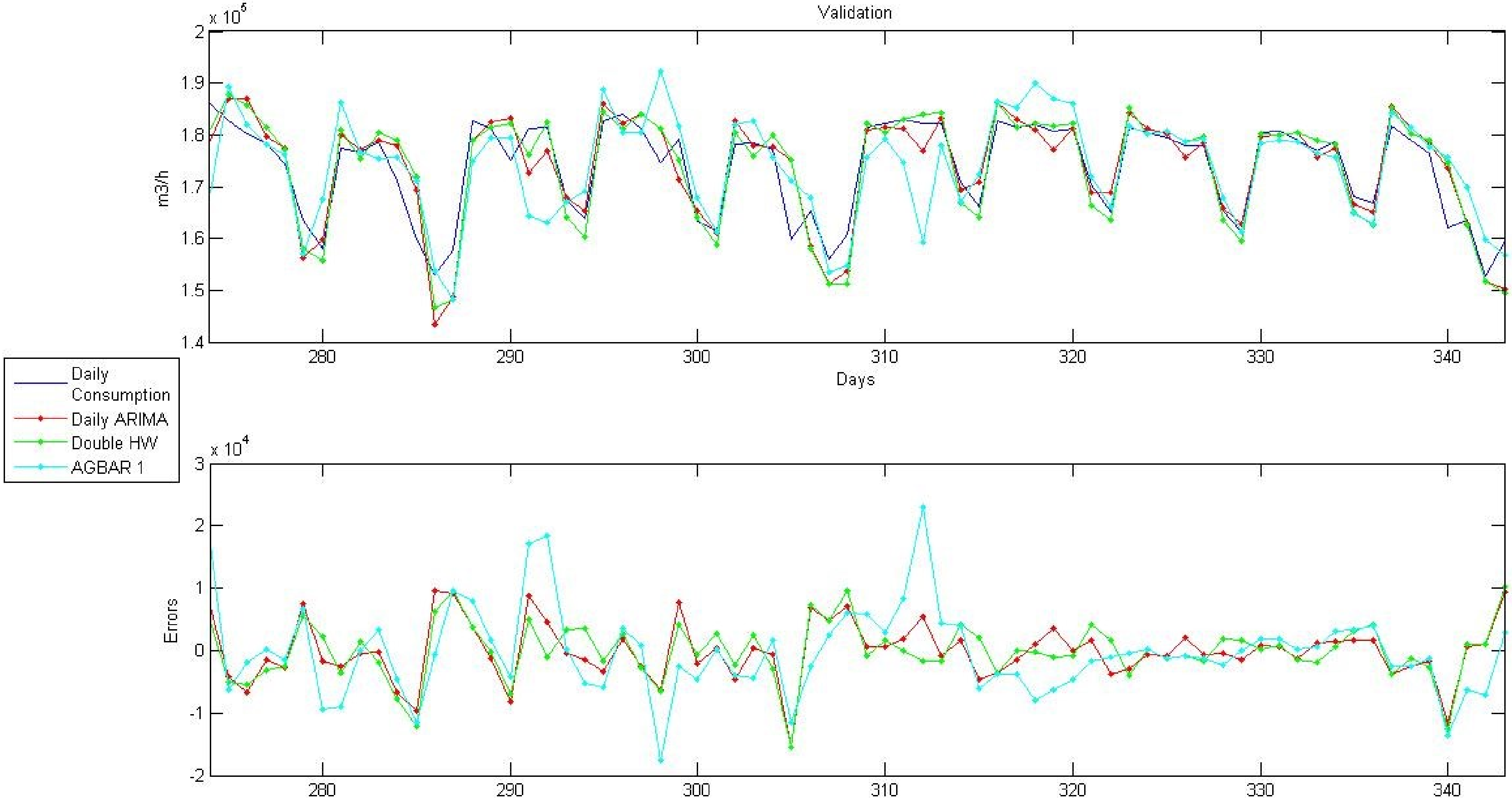
Ocampo-Martinez, C.; Puig, V.; Cembrano, M.; Quevedo, J. “Application of predictive control strategies to the management of complex networks in the urban water cycle”. IEEE Control Systems Magazine.33 - 1, pp. 15 -41. 2013. I SSN 1066-033X.

M. Brdys and B. Ulanicki, *Operational Control of Water Systems: Structures, algorithms and applications. UK: Prentice Hall International,* 1994

Water Demand Model

- The demand forecast module is needed for the MPC controller.
- Water demands presents two main seasonalities: hourly and weekly.
- Four methods have been studied: AGBAR methods, ARIMA models, basic structure models and Holt-Winters (HW) methods.
- Water demand has been characterized both daily and hourly.
- Water demand has been characterized at two levels: for each pressure floor and for the whole network.

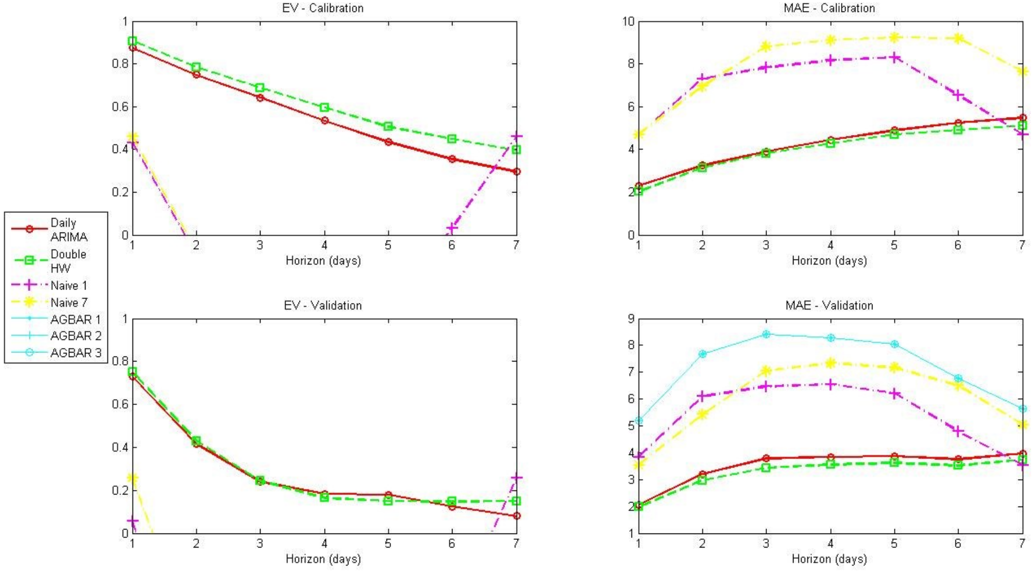
Daily Demand Model (1)



Daily Demand Model (2)

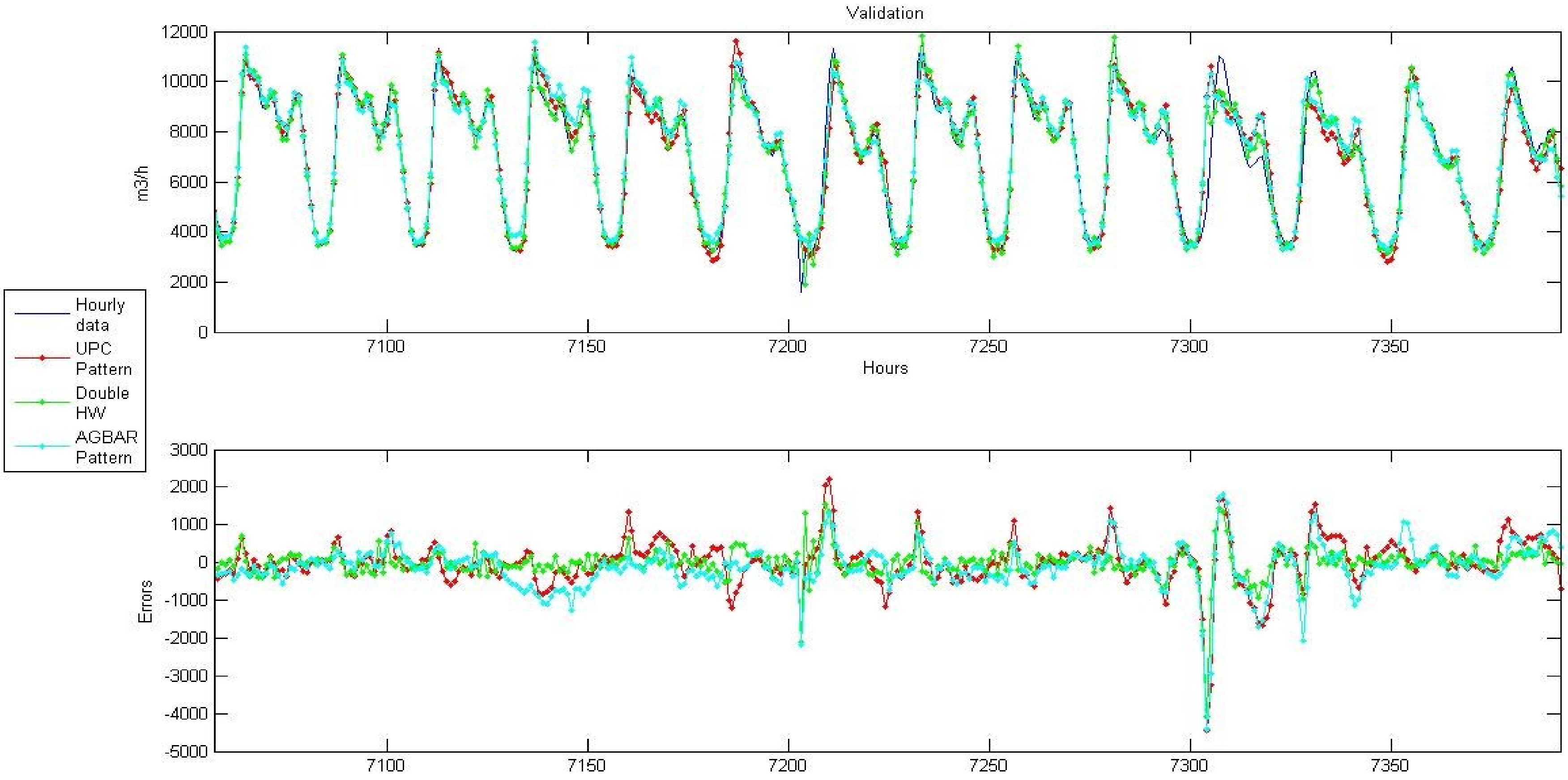
Conclusions:

The best forecast method is the double HW. The average absolute error of the double HW is considerably smaller than that of the AGBAR methods.



Horizonte	*ARIMA*	Holt-Winters	Naive1	Naive7	AGBAR1	AGBAR2	AGBAR3
1	2.6004	2.4168	3.9869	4.2652	5.1496	5.1271	5.1478
2	3.2734	3.1248	5.6994	5.6179	5.4208	5.4074	5.4193
3	3.5512	3.4308	6.2787	6.6960	5.4830	5.4774	5.4818
4	3.8580	3.7022	6.5007	7.1117	5.9926	5.9939	5.9921
5	4.2097	4.0945	6.5703	7.3217	6.4459	6.4491	6.4459
6	4.3269	4.2310	5.4119	7.2524	6.8670	6.8722	6.8671
7	4.4225	4.3377	4.2652	6.1098	7.2398	7.2483	7.2401

Hourly Demand Model (1)



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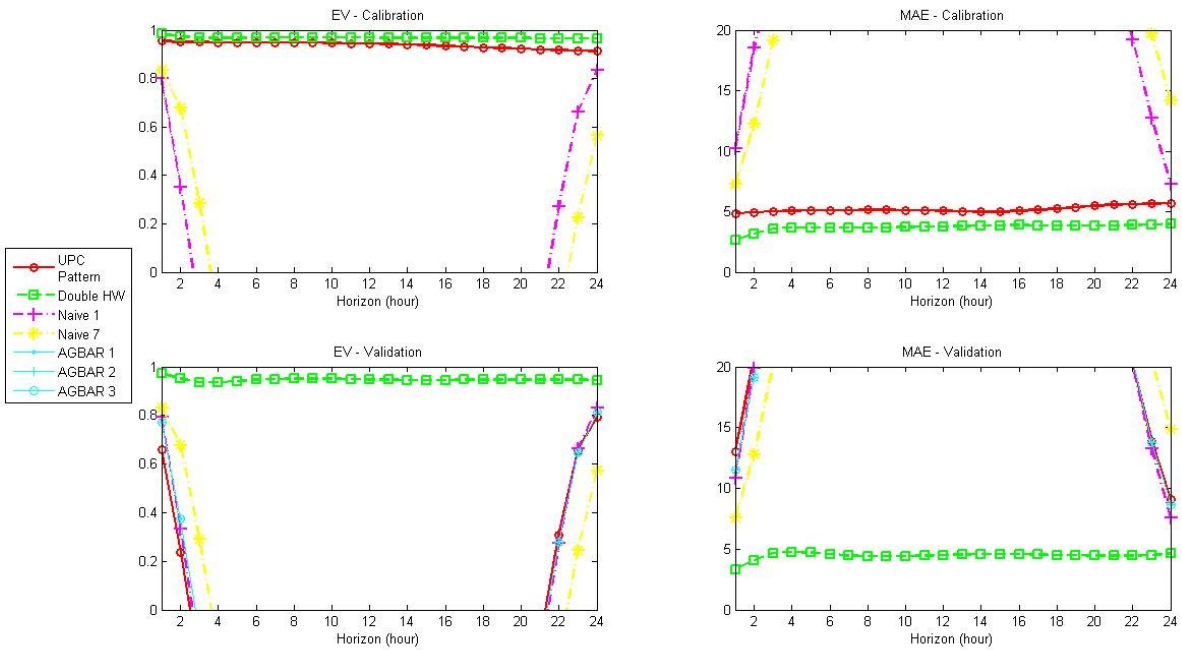
Hourly Demand Model (2)

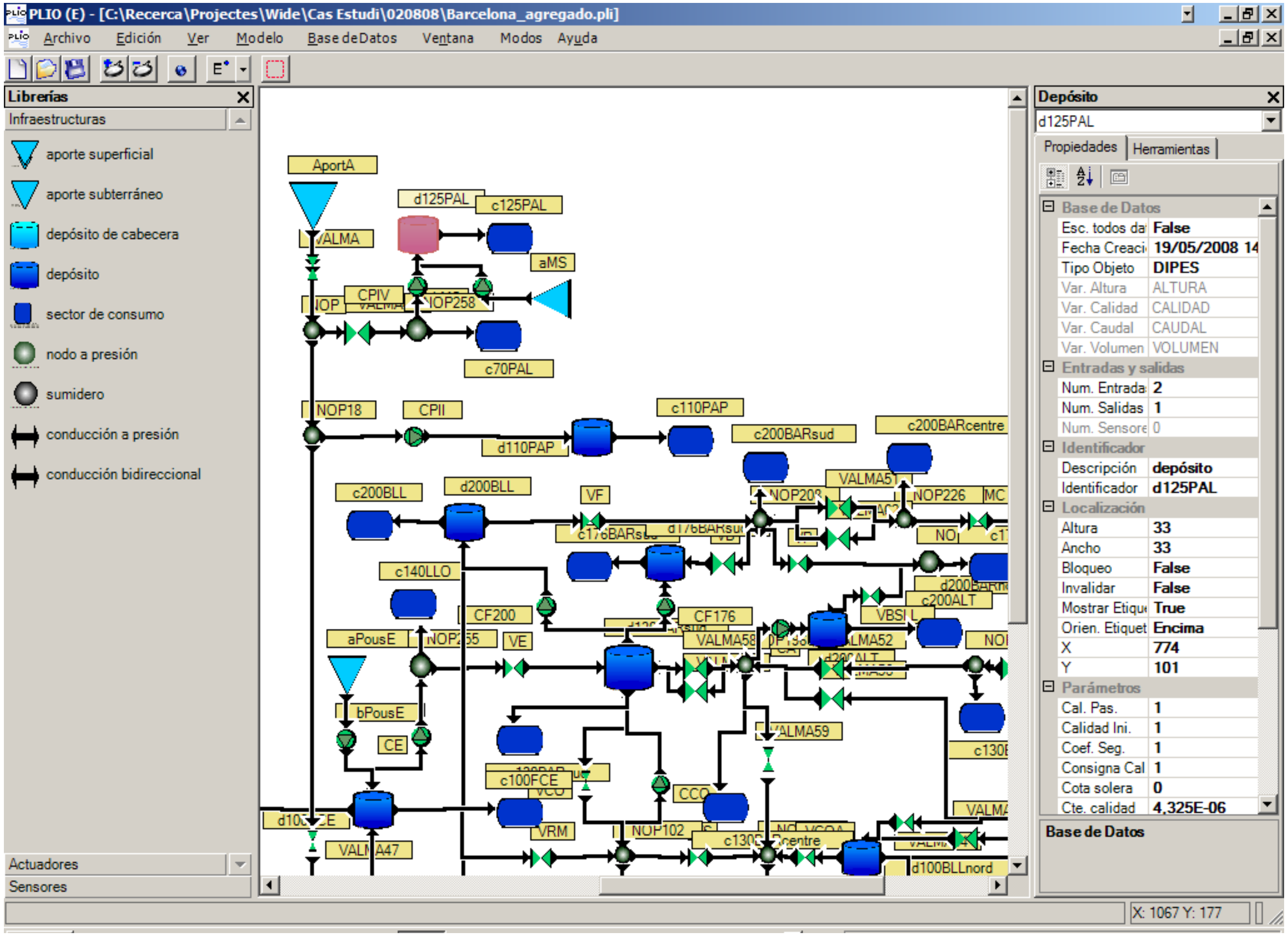
Conclusions:
The best forecast method is the double HW. The average absolute error of the double HW improves in comparison to the error of the AGBAR methods.



Horizonte	*ARIMA*	Holt-Winters	Naive1	Naive7	AGBAR1	AGBAR2	AGBAR3
1	5.7573	4.7620	10.8499	8.2796	16.1575	16.1602	16.1576
2	5.8429	5.0751	18.2944	12.3703	16.1666	16.1693	16.1667
3	5.9044	5.1949	24.6422	18.7919	16.1758	16.1786	16.1759
4	5.9505	5.2280	30.0094	24.9268	16.1846	16.1873	16.1847
6	6.0041	5.2305	37.5537	34.6366	16.1992	16.2018	16.1992
12	6.0369	5.2934	42.9574	43.8027	16.2625	16.2661	16.2627
24	6.4392	5.4075	8.2934	13.9050	16.4655	16.4695	16.4657

MPC Implementation
and
Validation on a Simulator

PLIO: MPC Cotrol of Water Networks



Cembrano, M.; Quevedo, J.; Puig, V.; .PLIO: a generic tool for real-time operational predictive optimal control of water networks. Water science and technology.64 - 2,pp. 448 - 459. 07/2011 .ISSN 0273-1223, 1994

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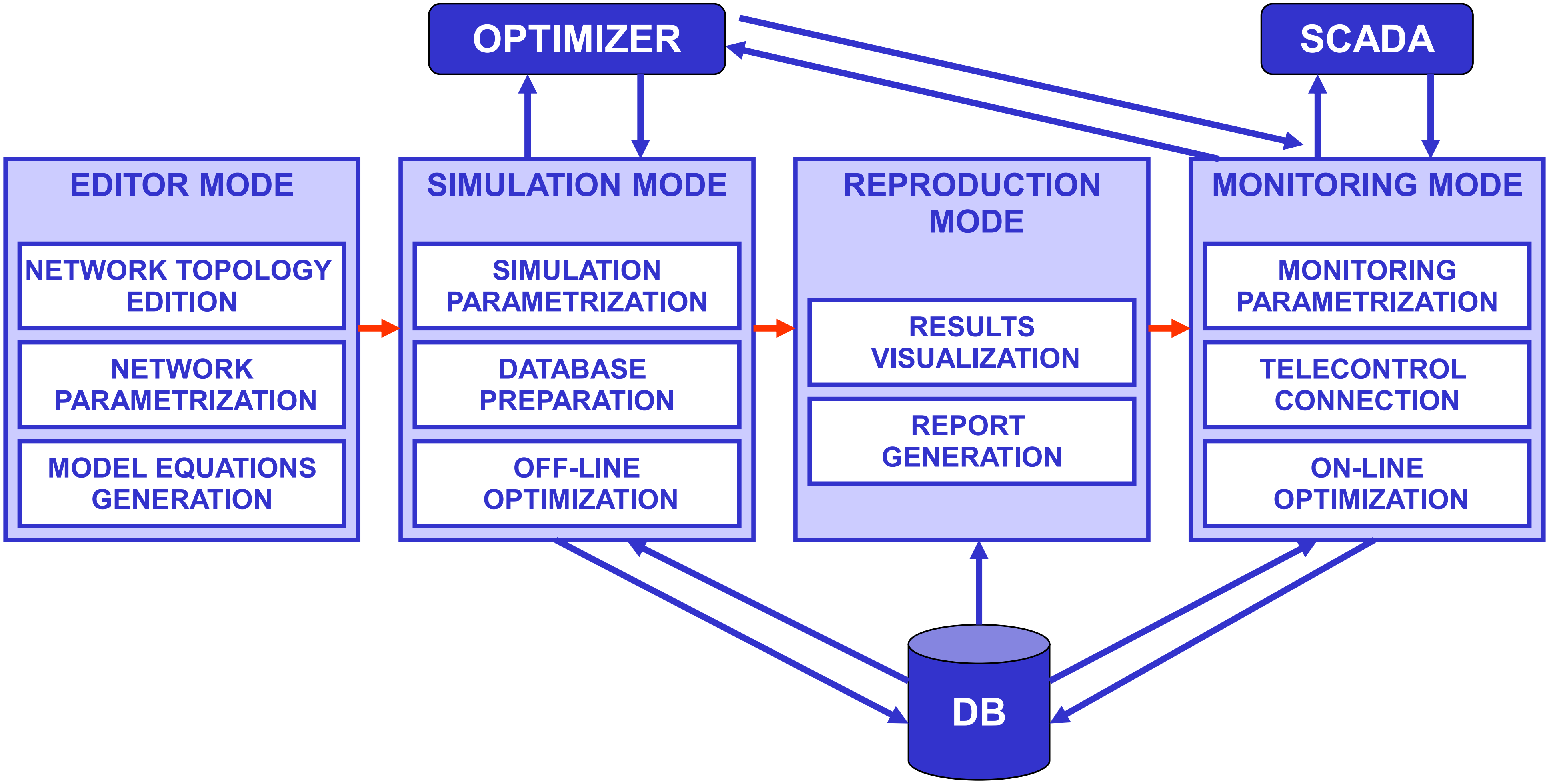
PLIO Architecture



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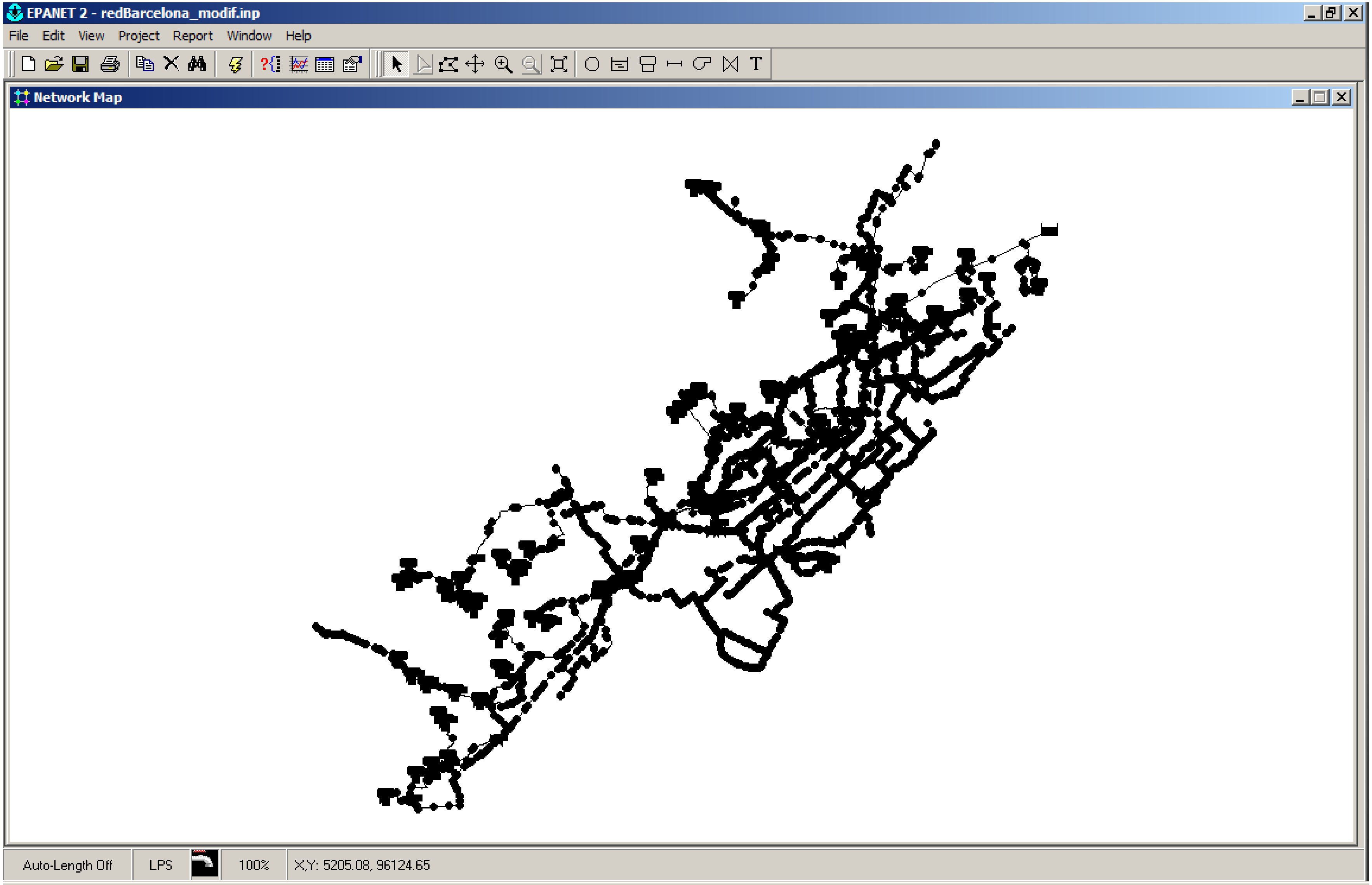
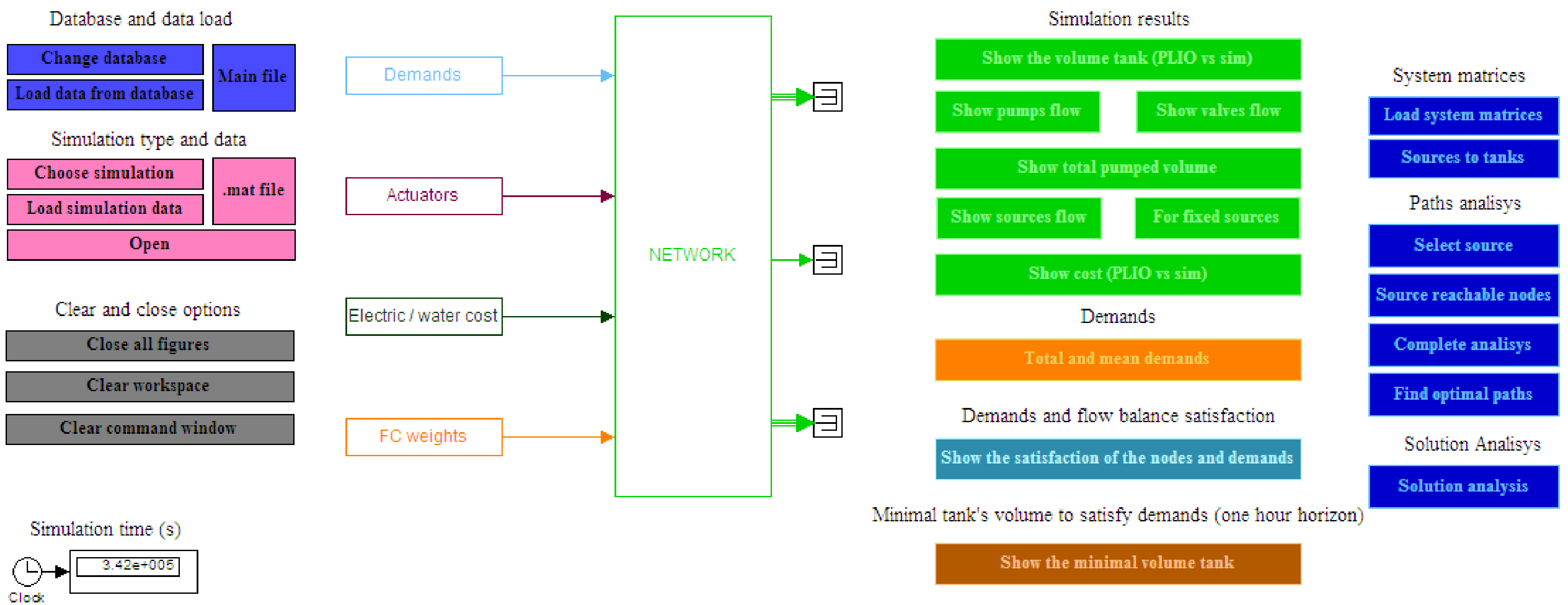
EPANET: Simulation of Water Networks

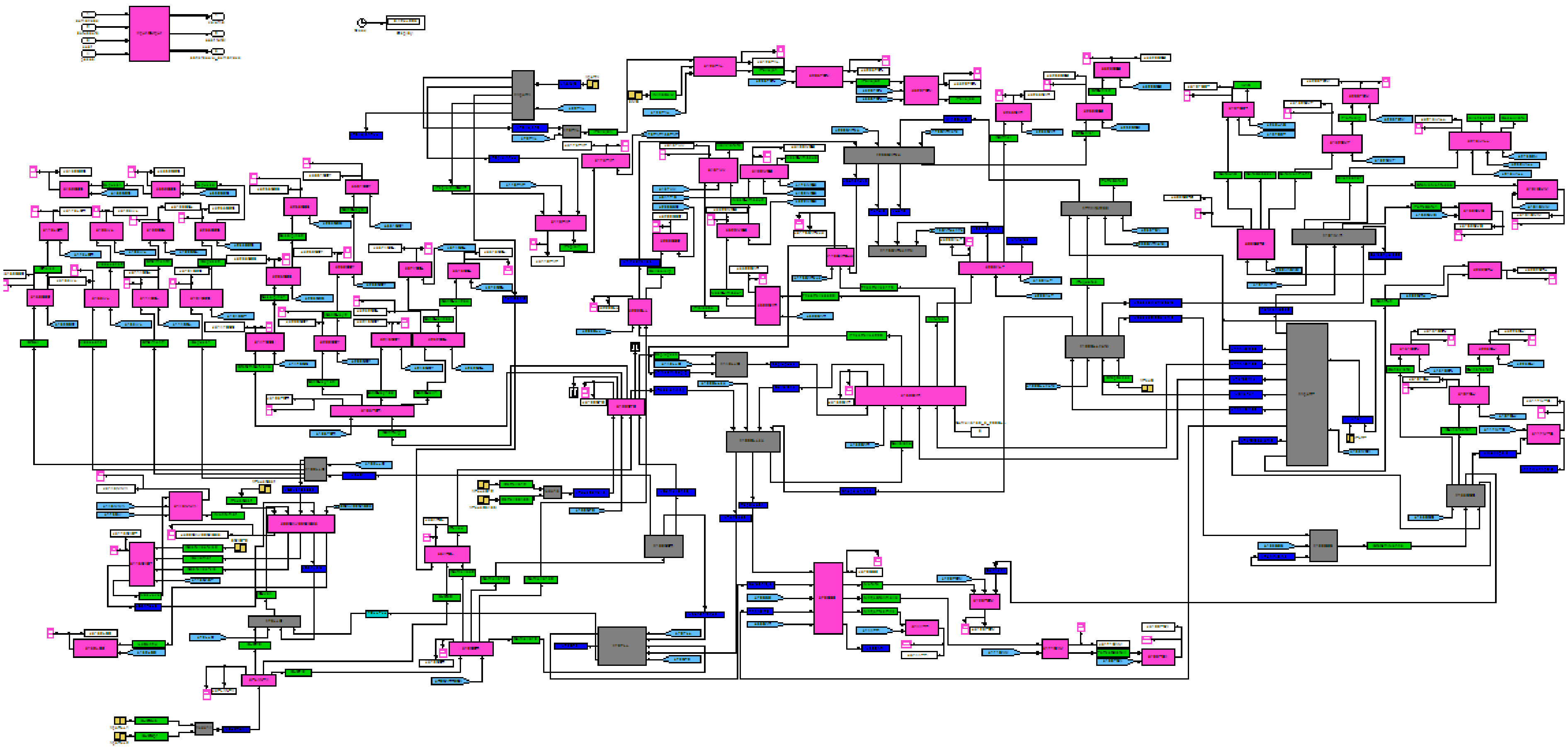


EPANET. *http://www.epa.gov/nrmrl/wswrd/dw/epanet.html*

Barcelona Network Simulator (1)



Barcelona Network Simulator (2)



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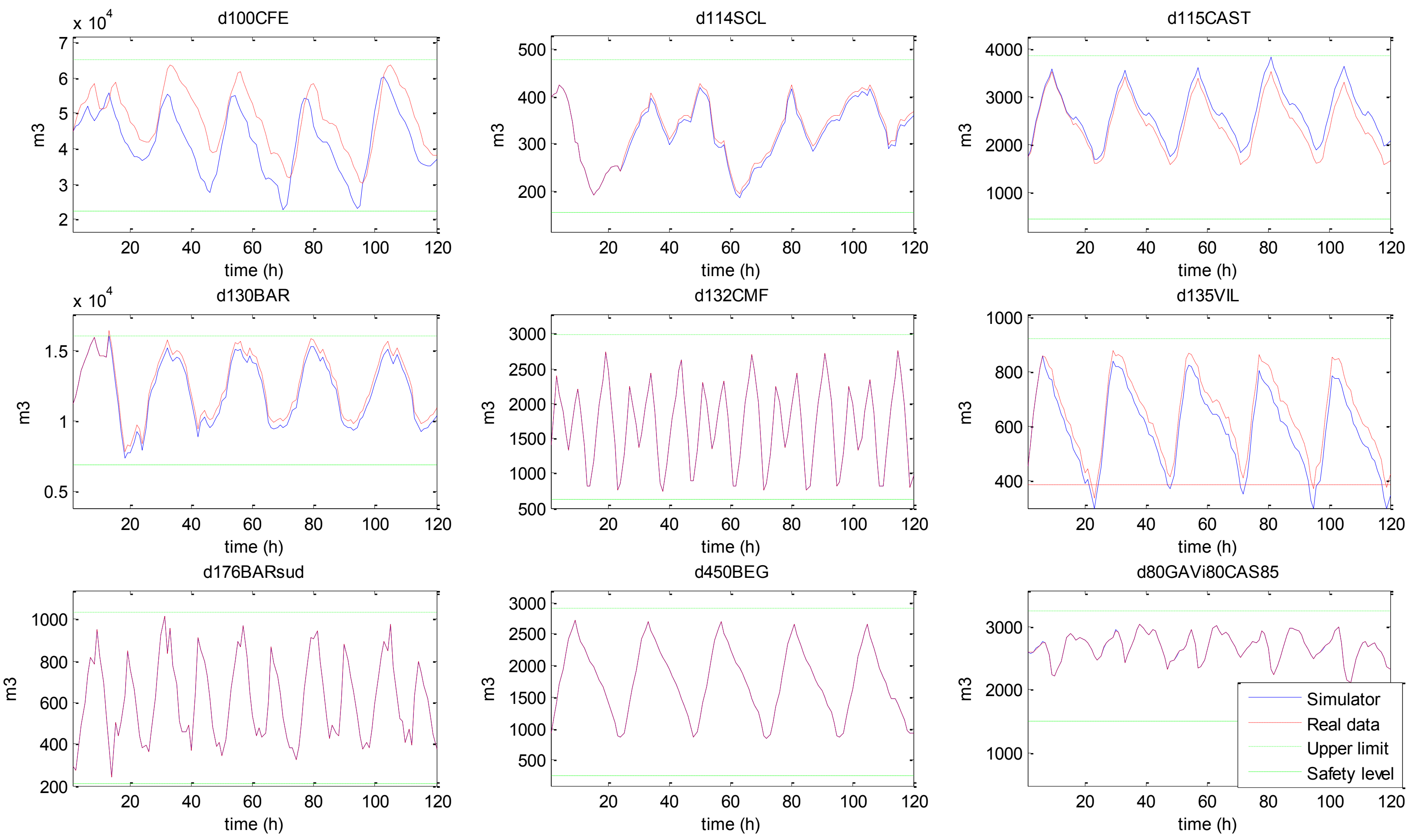
Model Validation against Real Data

Tank volumes comparison: model (blue) vs real (red).



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Results of MPC Control of the Barcelona Water Transport Network

MPC Results (1)

- MPC Control of Barcelona water network has been implemented by means of PLIO tool.
- To test and adjust the MPC controller some different scenarios have been studied. Parameters to take into account in the calibration of the model are:
  - Initial and security levels in tanks
  - Objective function weights: economical, safety and maintenance factors.
  - Working with different sources operation:
    - Llobregat source set at constant flow (Scenario 1)
    - Fixed sources at real flow (Scenario 2)
    - Source optimization. The optimizer calculates the flow for each time step inside the operational limits of each source (Scenario 3)

MPC Results (2)

Scenario 1: Llobregat source set at constant flow

	Flow(m^3/s)	
	Case 1	Case 2
Llobregat surface source	3	0
Llobregat underground source	2	2

- Barcelona's average input flow is about 7.5 m^3/s.
- In case 1 an important part of the total demand is taken from Llobregat.
- In case 2 only a 25% of the total demand is taken from Llobregat. It is expected that an important part of the network consumption is going to be taken from Ter.
- These two scenarios are interesting from the point of view of the behaviour of the economical cost.

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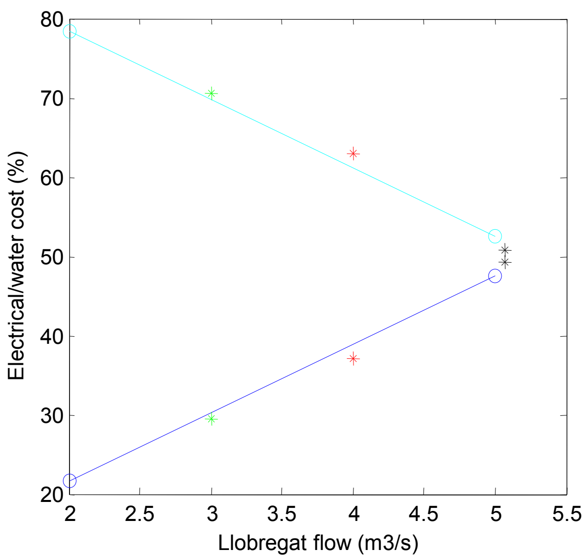
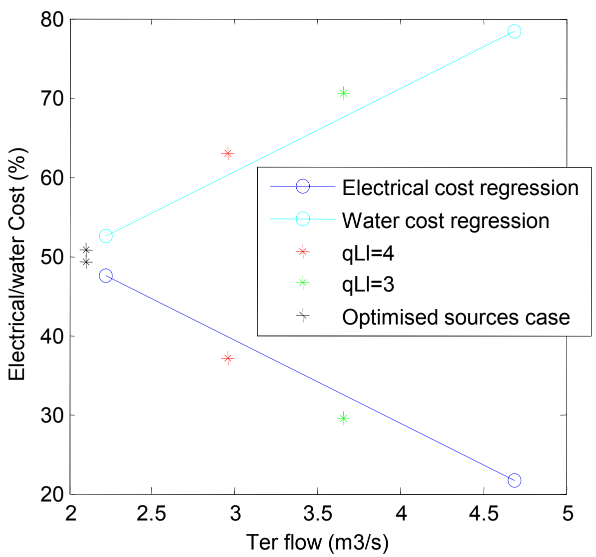
MPC Results (3)

- Conclusions
  - It exists a strong and linear dependency between economical cost and the operation of this two sources.
  - In order to reduce the total cost it is necessary to maximise the quantity of water taken from Llobregat.





Case 1

	Electrical cost	Water cost	Total cost
Day 1	52,42	47,58	100,00
Day 2	46,65	53,35	100,00
Day 3	48,10	51,90	100,00
Day 4	47,57	52,43	100,00

Case 2

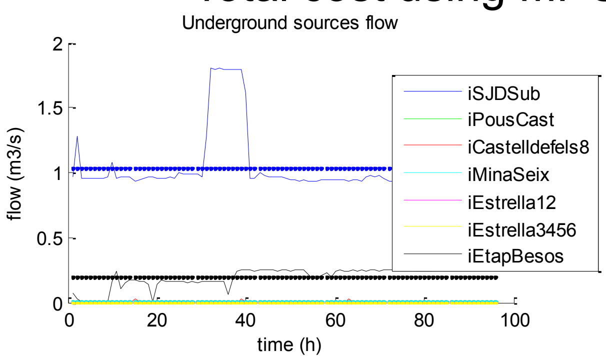
	Electrical cost	Water cost	Total cost
Day 1	-50,27	+91,34	+17,11
Day 2	-47,94	+72,77	+16,47
Day 3	-48,37	+78,27	+17,36
Day 4	-47,67	+71,06	+14,58

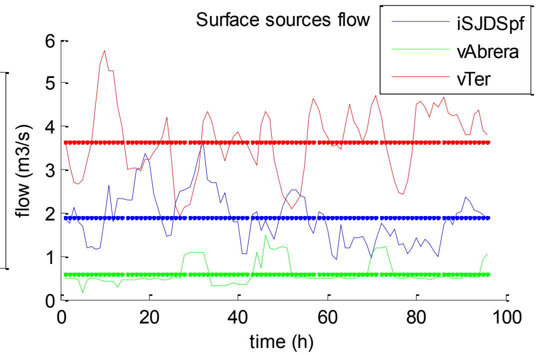
Increase/decrease % in comparison to case 1

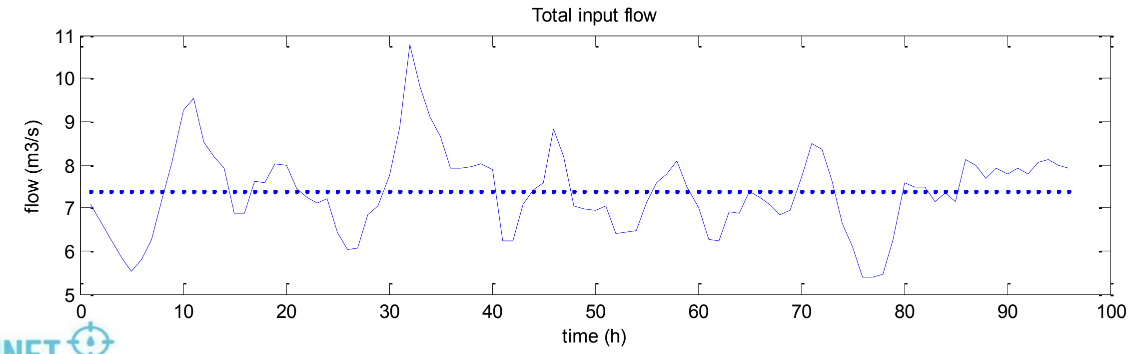
MPC Results (4)

Scenario 2: Sources set at real flow

- Sources flow is imposed by using real data obtained from AGBAR historical database.
- It is an interesting case study in order to compare centralised MPC control and current control applied regarding to transportation cost.
- It is a previous step before comparing centralised and decentralised MPC control.
- Important improvement in electrical cost, which represents between 10% and the 25 % of the real operation cost.
- Total cost using MPC control is between 4 and 8 % lower than the real one.







Current control

	Electrical cost	Water cost	Total cost
23/07/2007	33,13	66,87	100,00
24/07/2007	34,66	65,34	100,00
25/07/2007	32,00	68,00	100,00
26/07/2007	31,29	68,71	100,00

MPC

	Electrical cost	Water cost	Total cost
23/07/2007	-23,27	+0,00	-7,71
24/07/2007	-10,56	+0,00	-3,66
25/07/2007	-20,61	+0,00	-6,59
26/07/2007	-18,58	+0,00	-5,81

Increase/decrease % in comparison to current control

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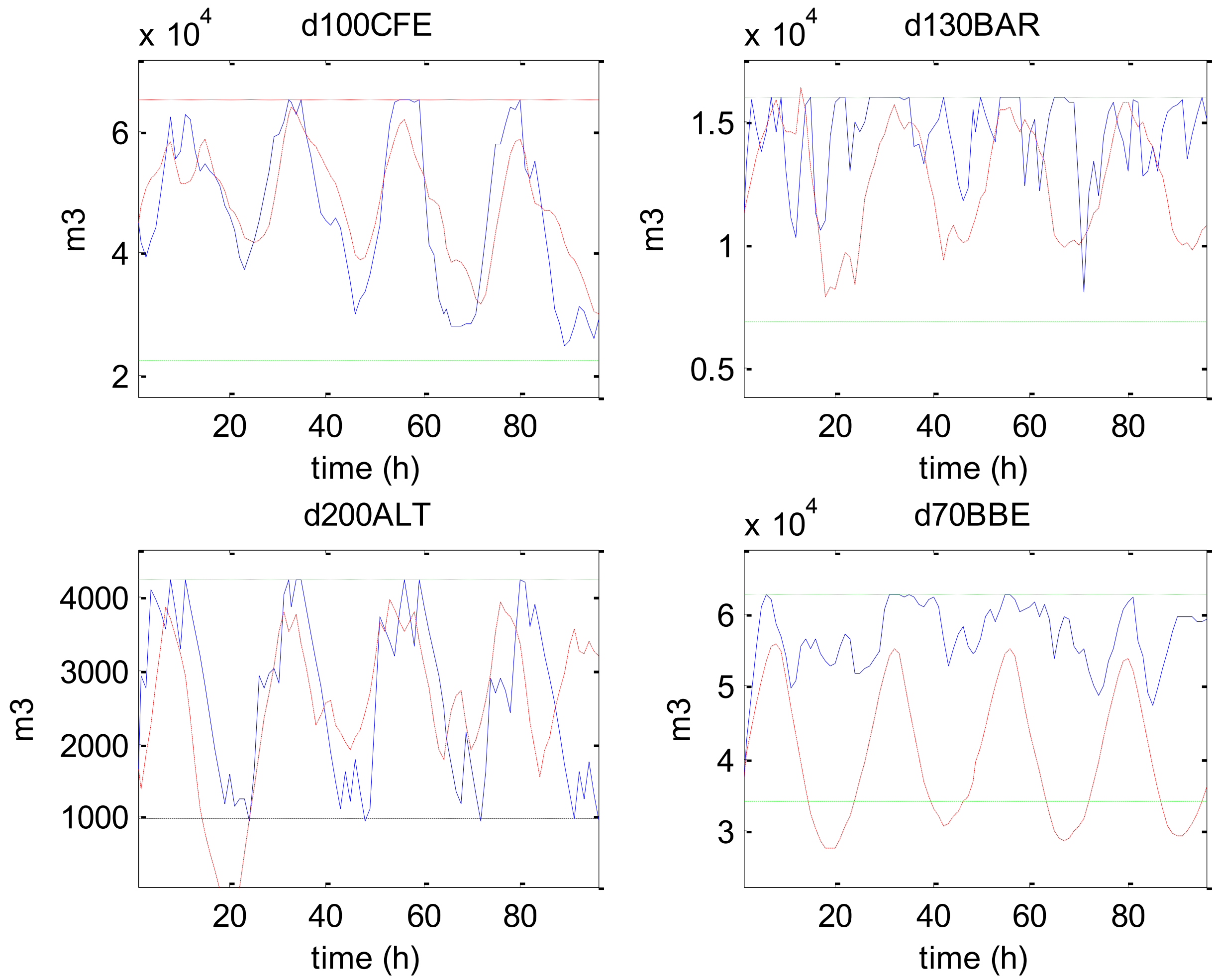
MPC Results (5)

- Some tanks volume evolution (real-red ,MPC-blue)



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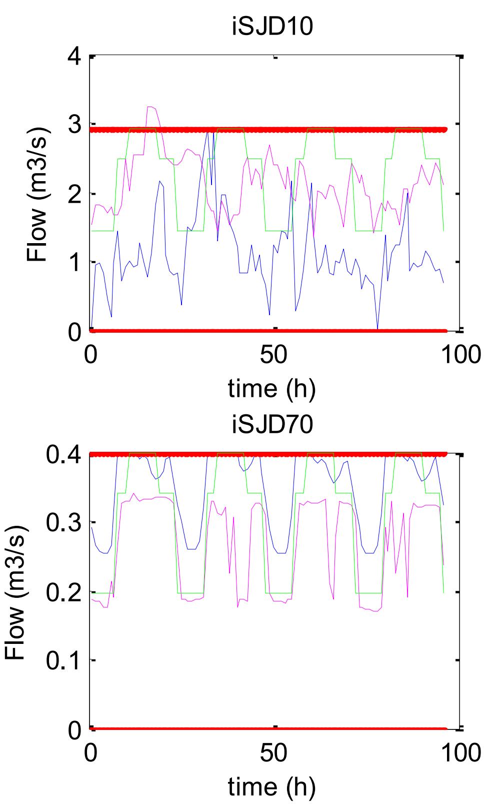
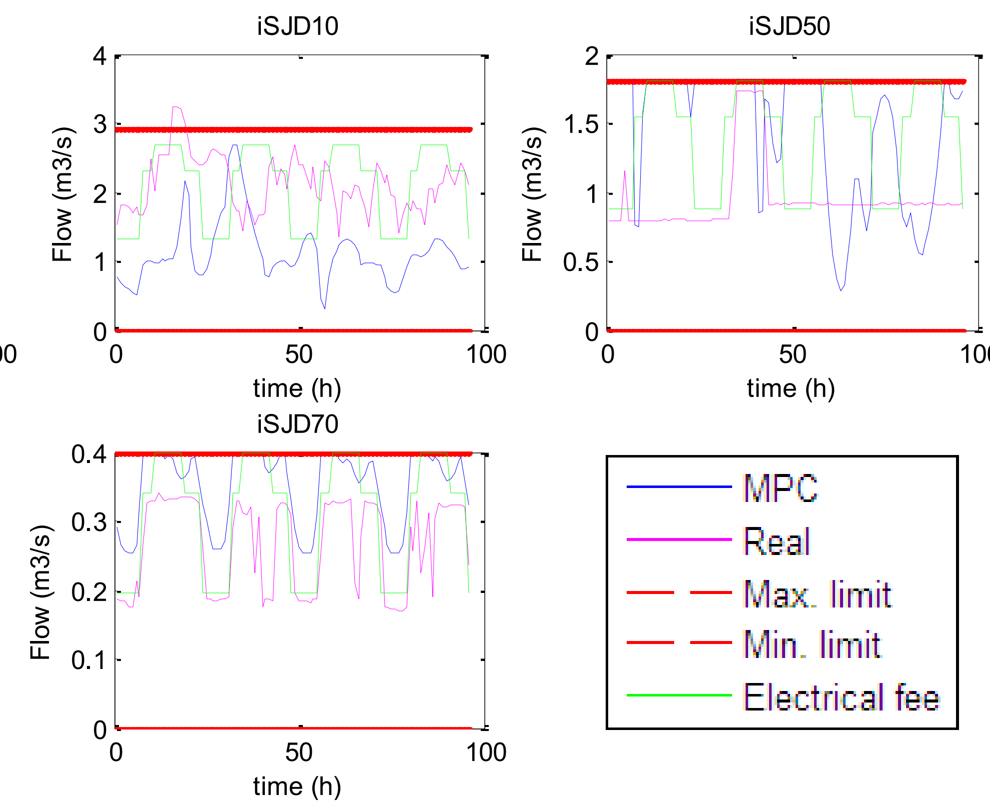
MPC Results (6)

- Stability term effects in pumps:

Without stability term

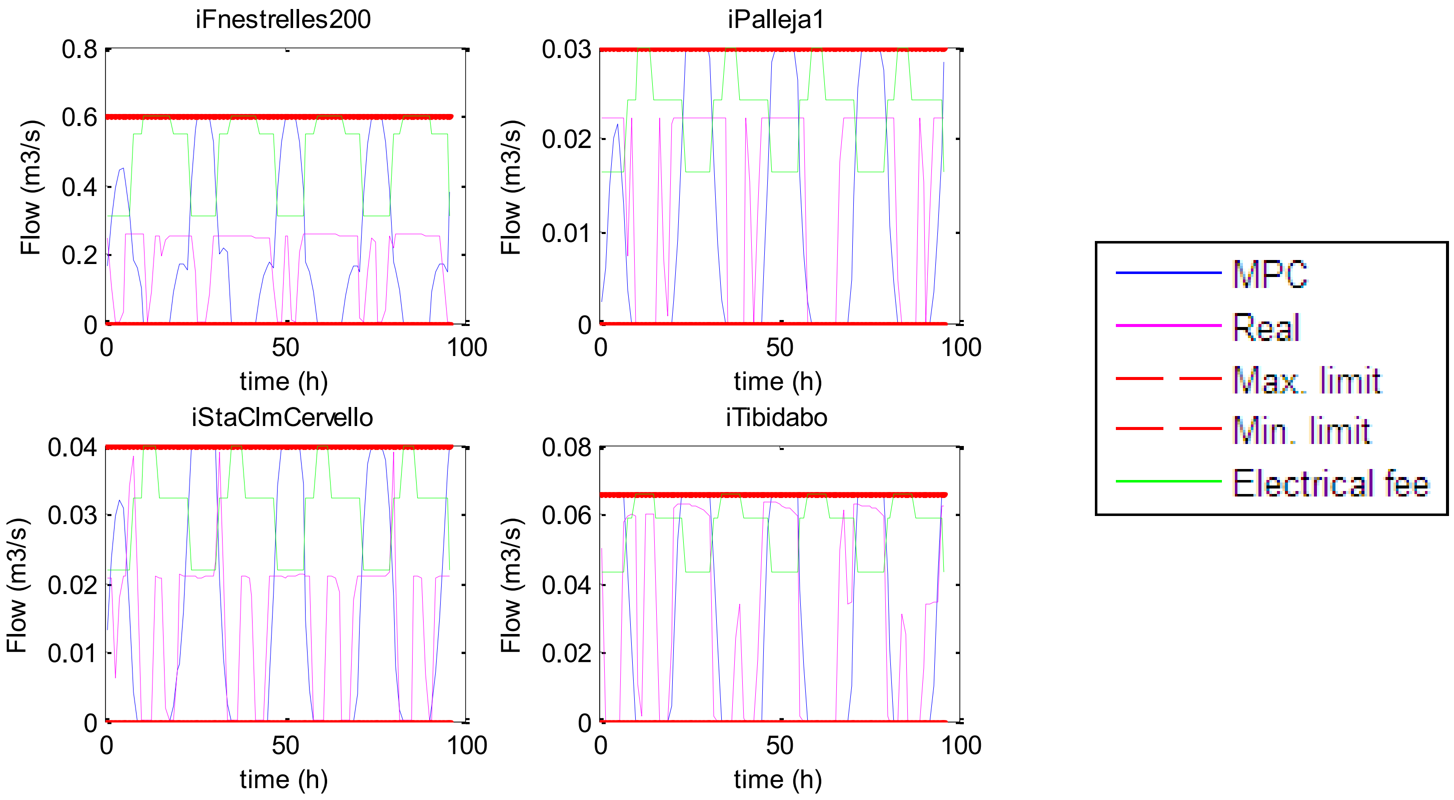


With stability term



MPC Results (7)

- Electrical cost depends on pumps operation. If it is possible pumps are only running during the cheapest period.



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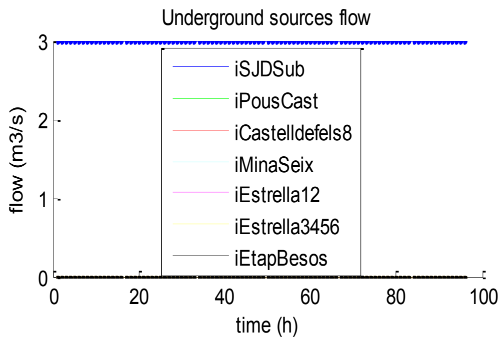
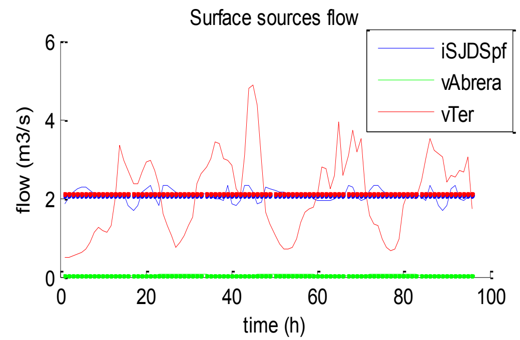
MPC Results (8)

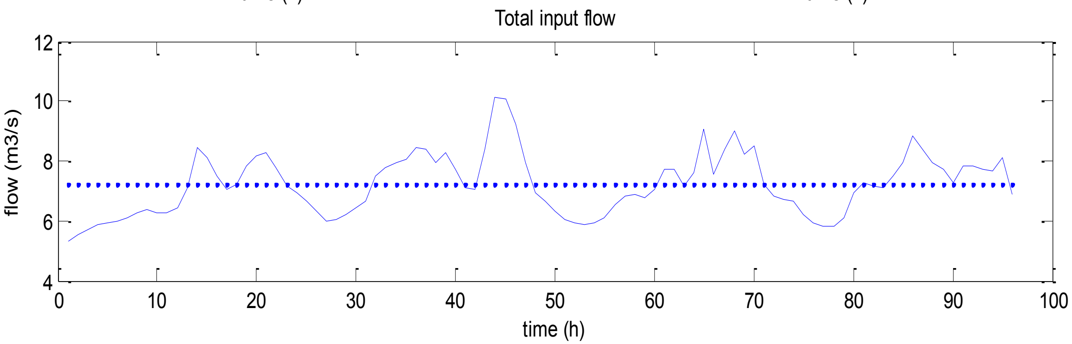
Scenario 3: Flow optimization

- In this case electrical and water costs are minimised, so it is expected a higher improvement in the total cost referring to the scenario with fixed sources.
- Taking into account results obtained in the first case study (constant fixed flow in Llobregat source) a solution with maximum average flow from Llobregat source is expected.
- In the optimization results shown the term that guarantees stability in control elements (pumps and valves) is on.
- Underground sources' water cost is penalized to avoid its over-exploitation.

MPC Results (9)







- Big water cost savings, between 30% and 50 %.
- Electrical cost has increased regarding to current control case ([+18,+27]%) and MPC case with fixed sources ([+27,+60]%).
- Total cost has decreased between 13% and 22 % regarding to MPC results obtained with fixed sources.
- Sources flow distribution is the expected one. Llobregat's source flow is maximized.

Current control

	Electrical cost	Water cost	Total cost
23/07/2007	33,13	66,87	100,00
24/07/2007	34,66	65,34	100,00
25/07/2007	32,00	68,00	100,00
26/07/2007	31,29	68,71	100,00

MPC improvement in comparison to current control case

	Electrical cost	Water cost	Total cost
23/07/2007	18,92	-50,70	-27,63
24/07/2007	14,04	-32,56	-16,41
25/07/2007	26,29	-43,91	-21,45
26/07/2007	26,09	-44,43	-22,36

MPC improvement in comparison to fixed sources to real flow case (Scenario 2)

	Electrical cost	Water cost	Total cost
23/07/2007	54,99	-50,70	-21,59
24/07/2007	27,51	-32,56	-13,23
25/07/2007	59,08	-43,91	-15,91
26/07/2007	54,86	-44,43	-17,57

Results Summary

Cost	Current control	Scenario 2 (electricity)	Scenario 3 (source/electricity)
Electrical	32,77%	-18,26%	+21,34%
Water	67,23%	0	-42,90%
Total	100%	-5,94%	-21,96%

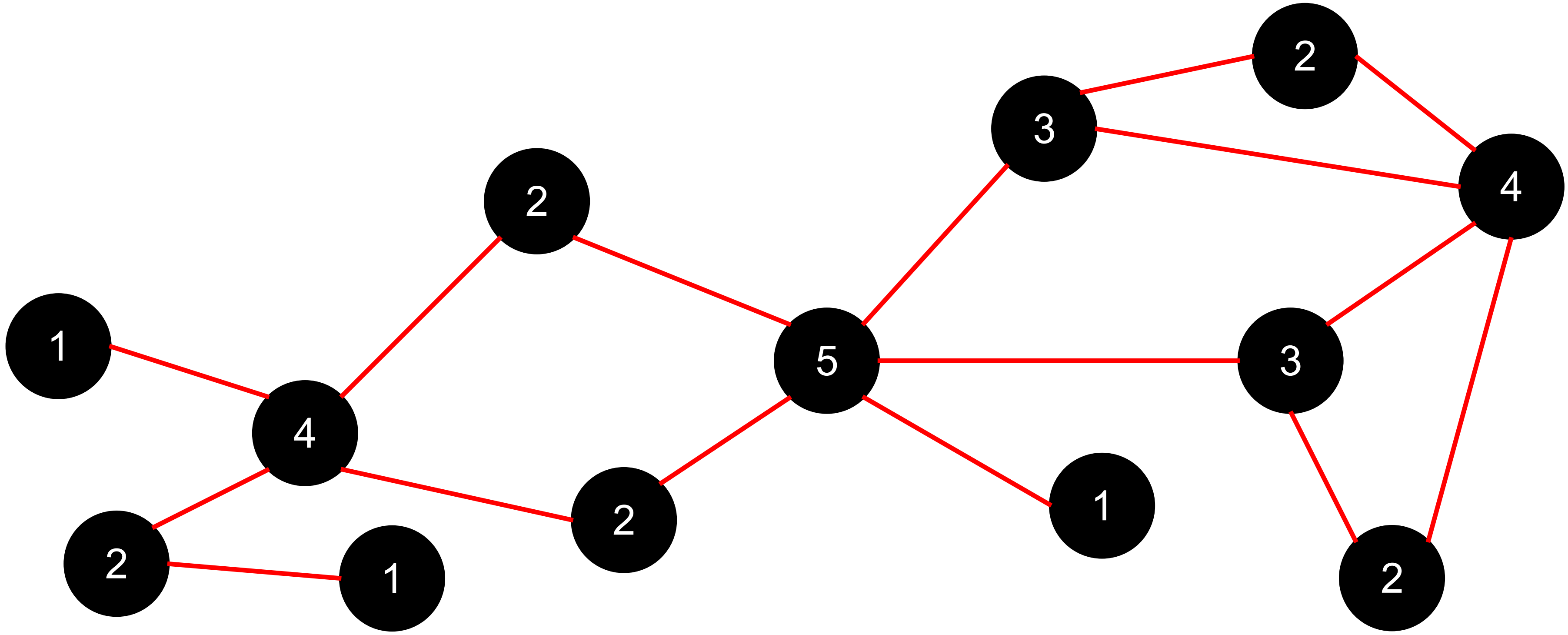
Decentralising/Distributing the MPC control in Water Networks

Partitioning Algorithm

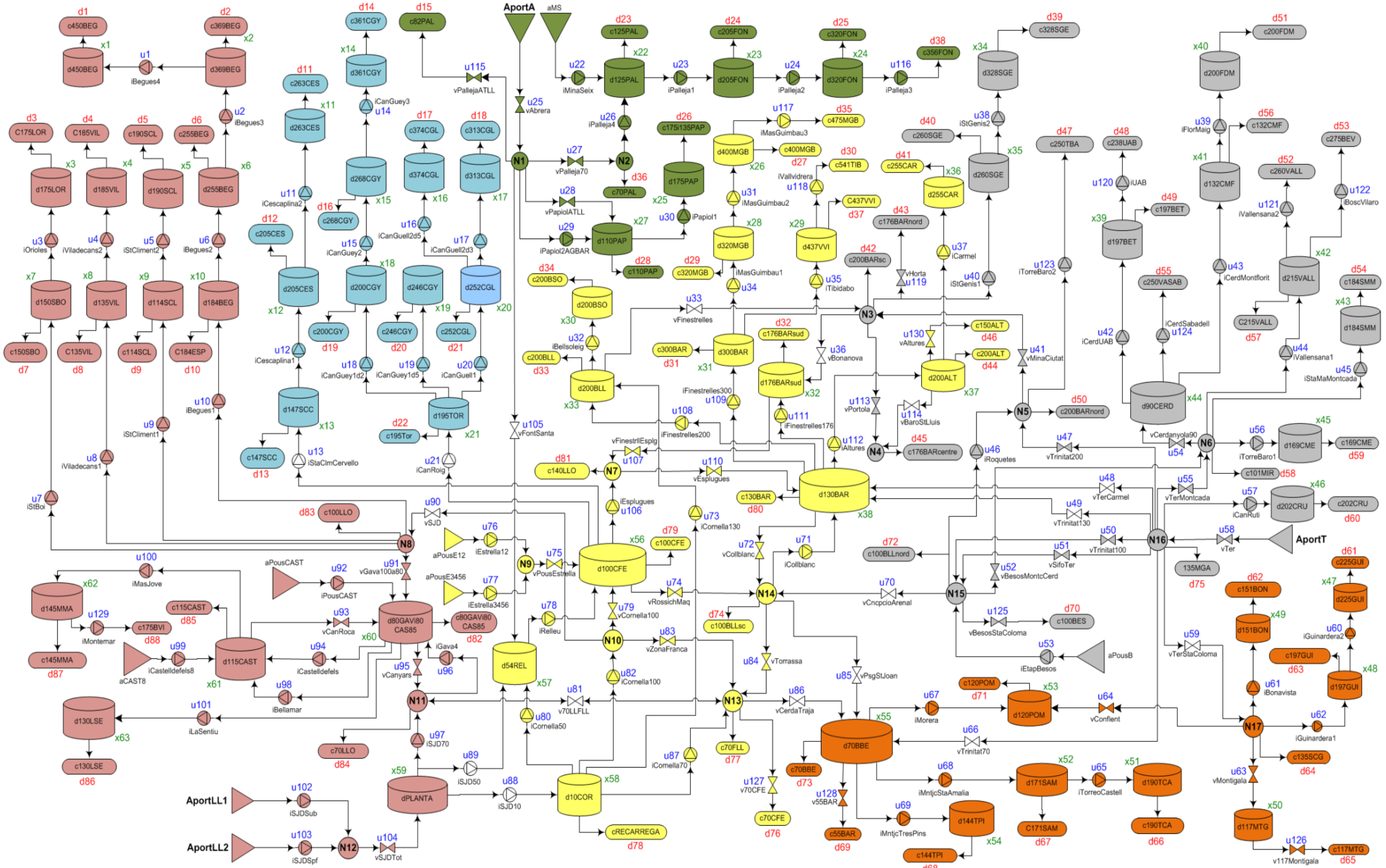
Algorithm Steps

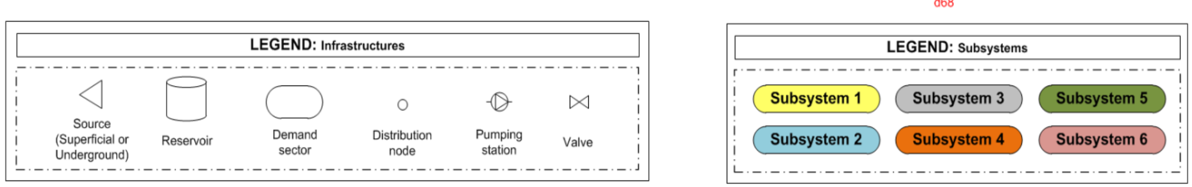
- ✓ Start up
- ✓ Preliminary partitioning
- ✓ Uncoarserning (Internal balance)
- ✓ Refining (External balance)
- ✓ Auxiliary routines (Pre/post-filtering)

C. Ocampo-Martinez, S. Bovo, V. Puig. Partitioning Approach oriented to the decentralised MPC of Large-Scale Systems. Journal of Process Control, 21(5):775-786, 2011.



Partitioning Results of Barcelona Network (1)





63	tanks
121	actuators
88	demands
15	nodes

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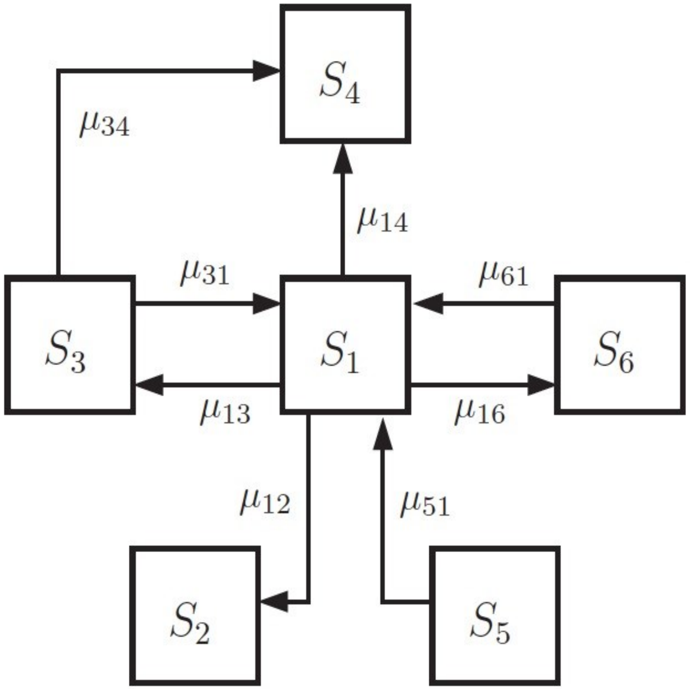
Partitioning Results of Barcelona Network (2)

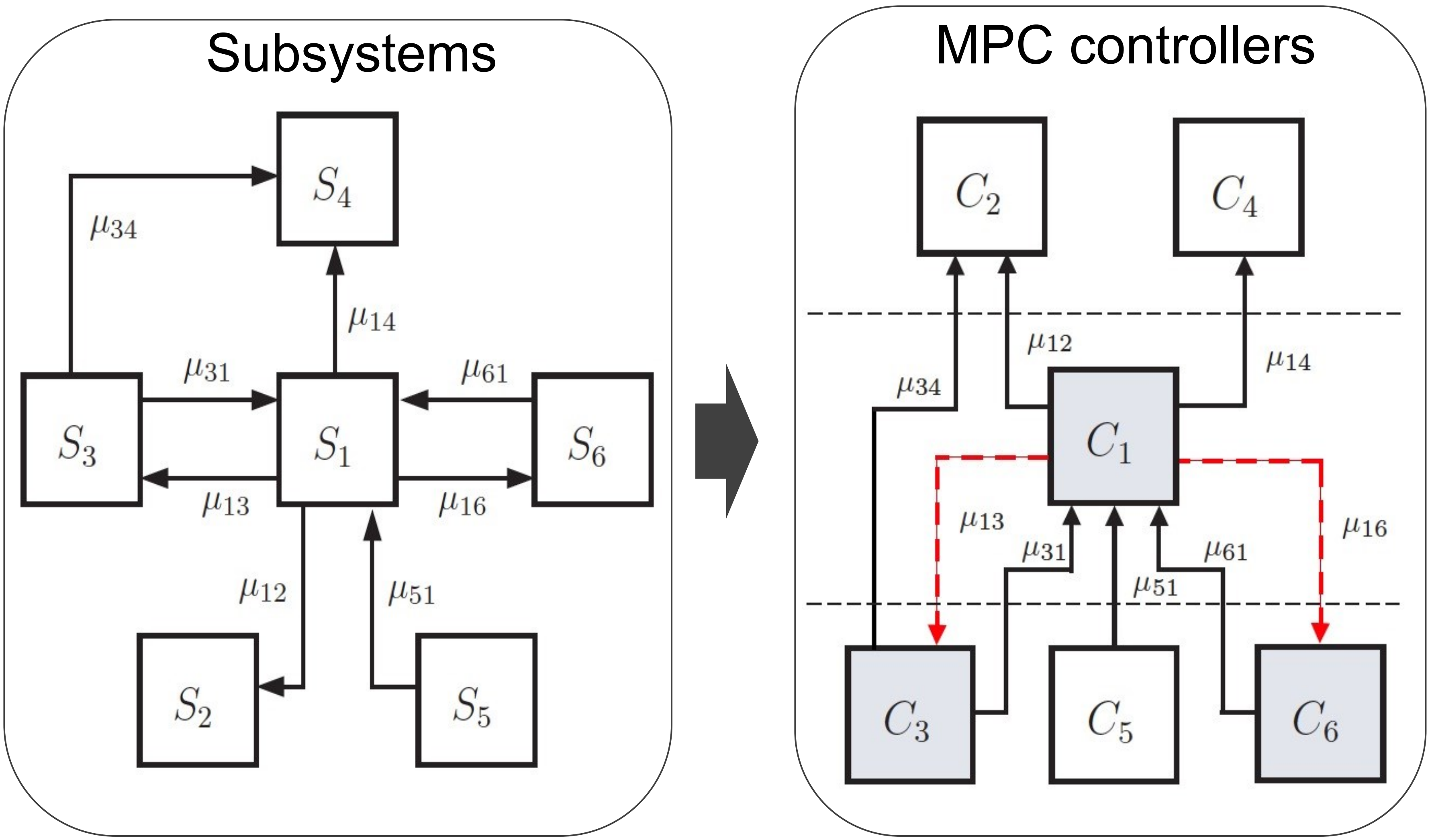
- ✓ every tank, sector of consume, water source and node is a vertex of the graph
- ✓ every pump, valve and link with a sector of consume is a graph edge



Subsystem	Tanks	Actuators	Demands	Nodes
1	13	36	20	5
2	11	11	11	0
3	13	22	20	3
4	9	16	12	2
5	6	10	8	2
6	15	26	17	3
Total	67	121	88	15

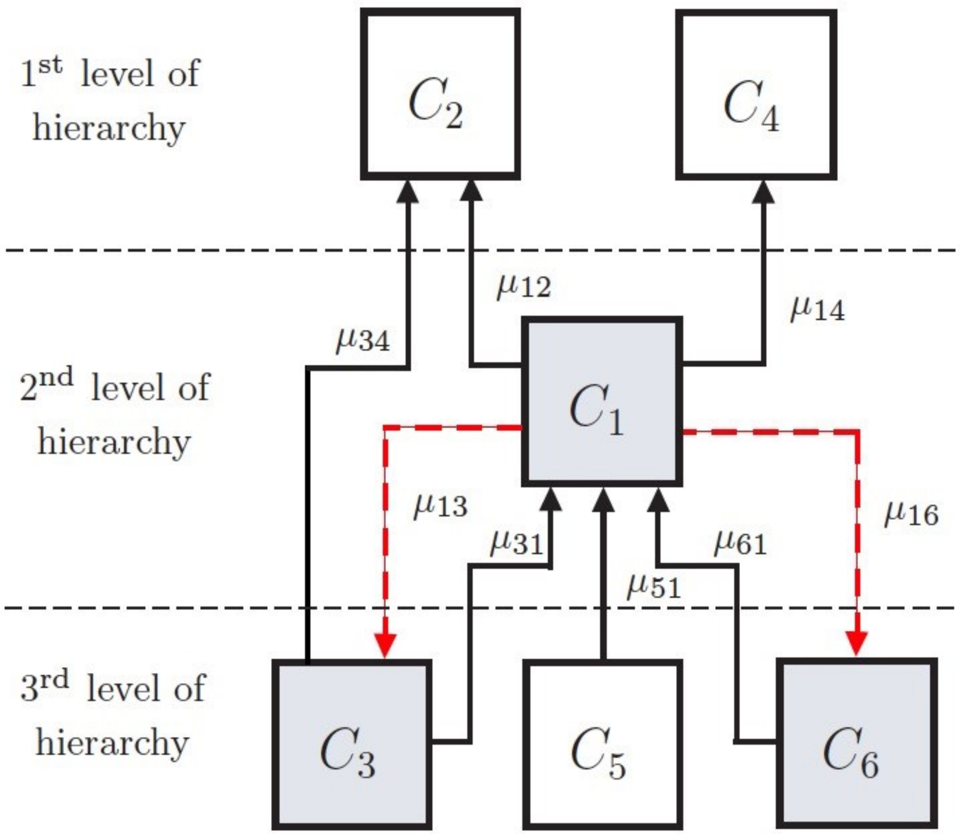
Comparison of the dimension of the resultant subsystems

Hierarchical-like DMPC Approach



Ocampo-Martinez, C.; Barcelli, D.; Puig, V.; Bemporad, A.Hierarchical and decentralised model predictive control of drinking water networks: Application to Barcelona case study.IET control theory and applications.6 - 1,pp. 62-71 2012.

Hierarchical-like DMPC Approach



SOLVING SEQUENCE

- ✓C_4 for S_4 and μ_{14}, μ_{34}.
- ✓In parallel, C_2 for S_2 and μ_{12}.
- ✓C_1 for S_1 and sets μ_{31}, μ_{51}, and μ_{61}. Sets μ_{12}, μ_{13}, μ_{14}, μ_{16} are *virtual demands (VD)* for C_1.
- ✓C_5 for S_5 with μ_{51} as VD.
- ✓C_3 for S_3 with μ_{31}, μ_{34} as VD. C_3 also computes μ_{13} as VD for C_1 in *t + 1*.
- ✓C_6 for S_6 with μ_{61} as VD. C_6 also computes μ_{16} as VD for C_1 at *t + 1*.

Values of μ_{13}, μ_{16} at t=1 ⟹ CSP with S1, S3 and S6

DWN Management Criteria

1

Minimizing water production and transport cost

$$J_1(k) = (\alpha_1 + \alpha_2(k))u(k)$$

Cost of water at source
(water taxes and treatment costs)
GLOBAL OBJECTIVE

Cost of water transport
(mainly due to pumpung costs)
LOCAL OBJECTIVE

2

Ensuring safety water storage

$$J_2(k) = \epsilon(k)^T \epsilon(k)$$

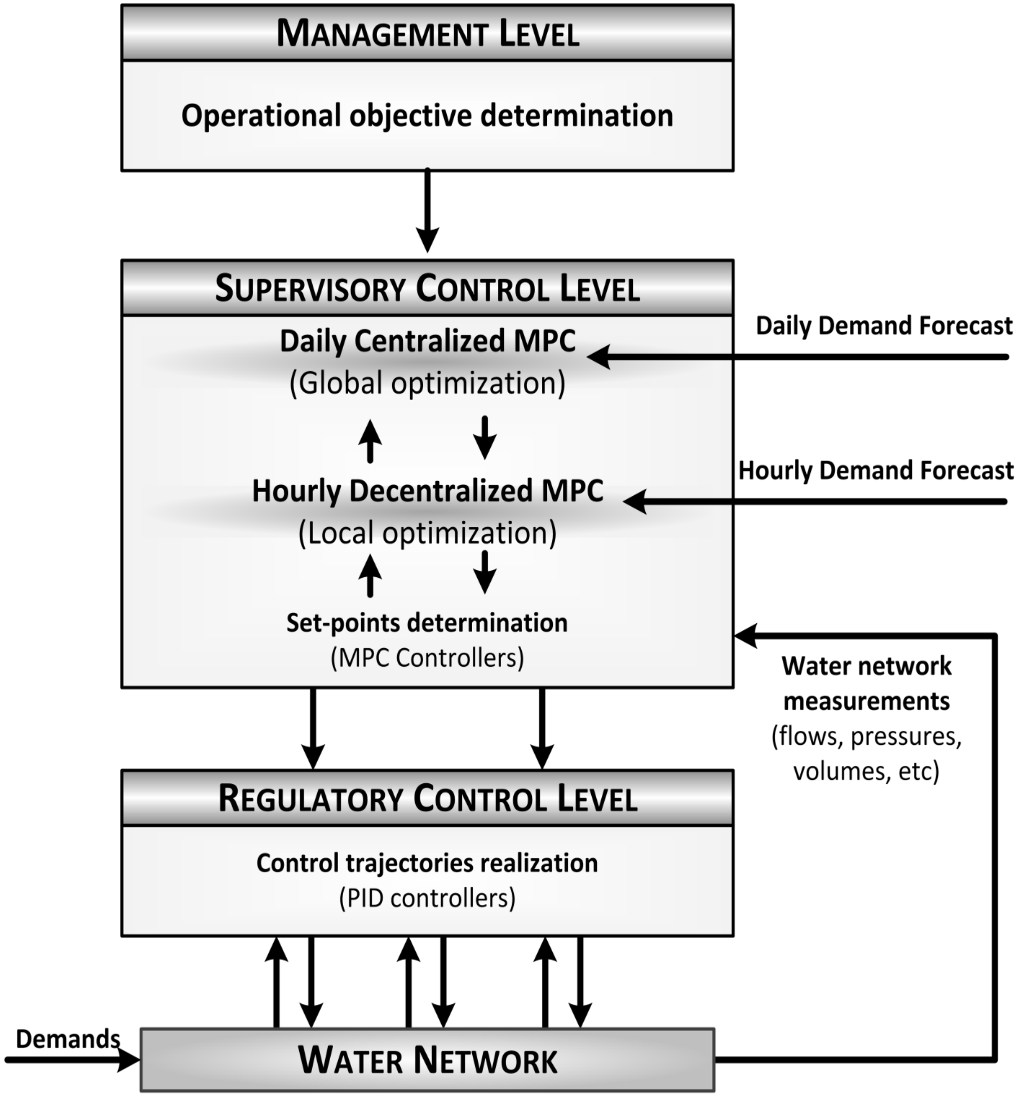
3

Ensuring smoothness of the control actions

$$J_3(k) = \Delta u(k)^T \Delta u(k)$$

Multi-temporal DMPC

C. Ocampo-Martinez, V. Puig, J.M. Grosso and S. Montes-de-Oca
Multi-layer Decentralized Model Predictive Control of Large-Scale Networked Systems. Distributed MPC made easy. Springer. 2013.



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Main Results

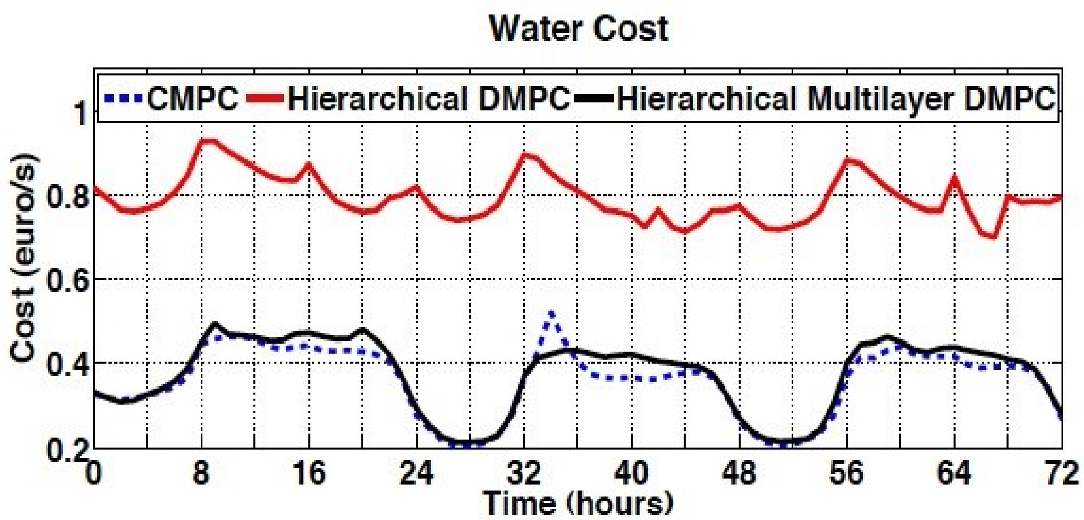
Economic costs (Performance comparisons)

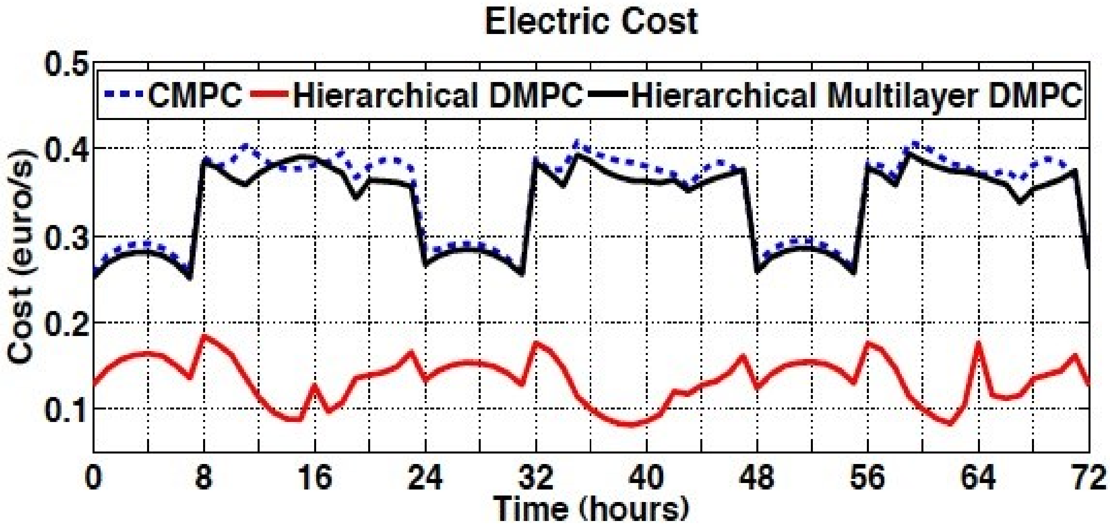
INDEX	CMPC	DMPC	ML-DMPC
Water Cost	93.01	205.55	97.11
Electric Cost	90.31	34.58	87.53
Total Cost	183.33	240.13	184.65

MATLAB® 7.1, Intel® Core™2, 2.4 GHz, 4Gb RAM

Economic units (due to confidenciality reasons)

Main Results: Costs





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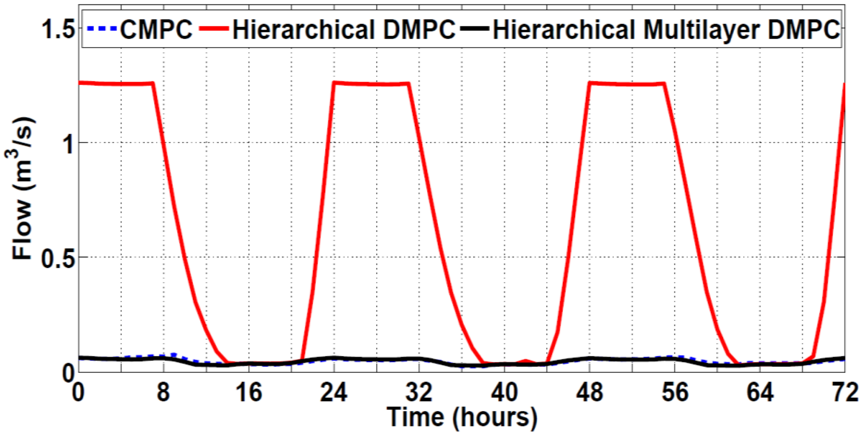
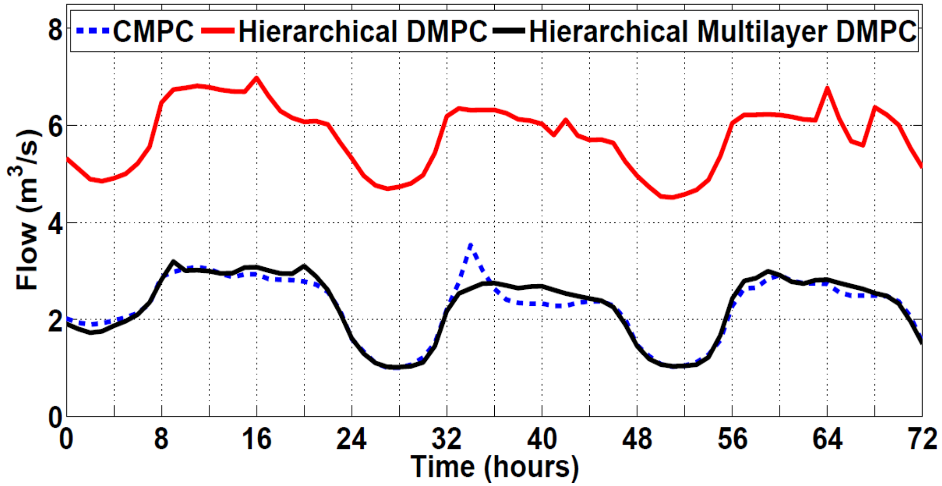
UPC

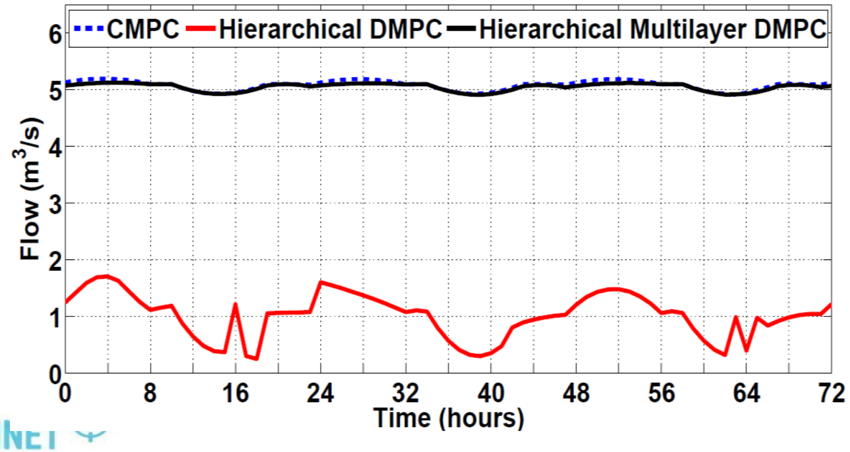
Main Results: Inflows (sources)



Abrera Flow

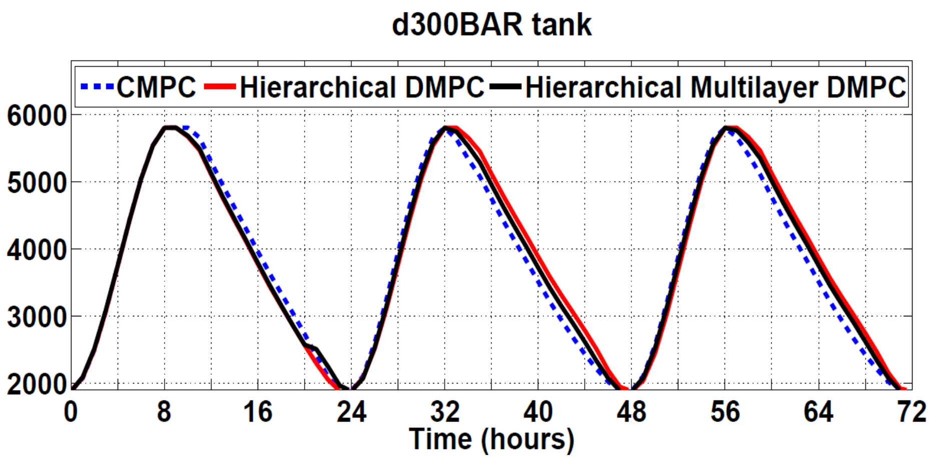
Ter Flow

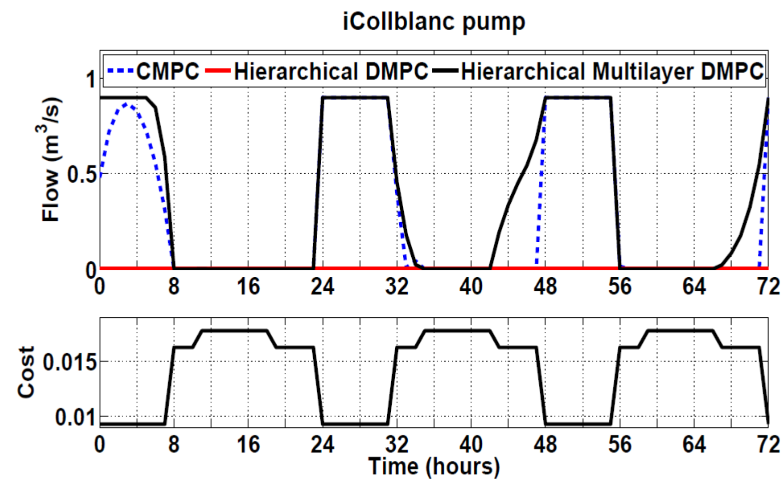


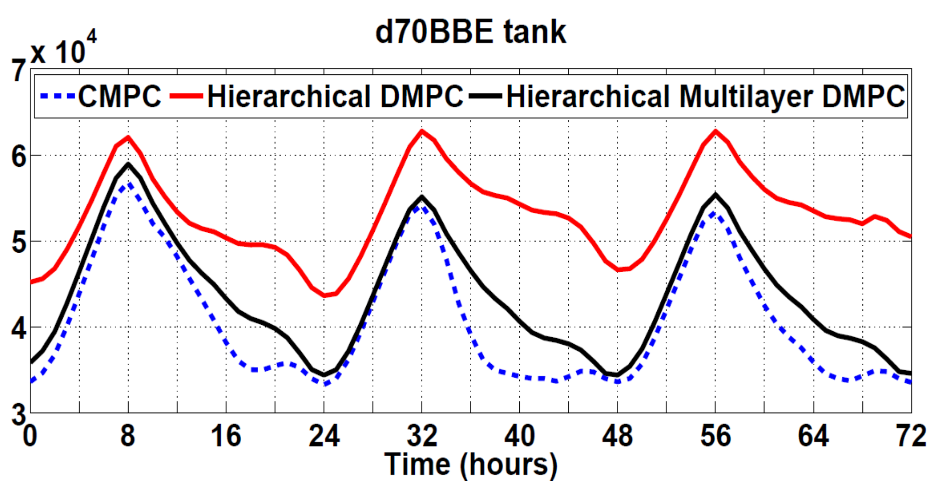


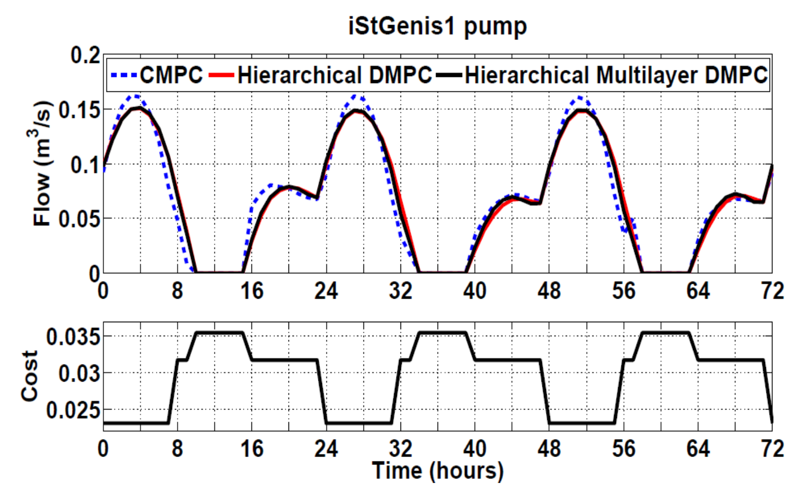
Llobregat Flow

Main Results: Behaviour in Elements









MPC of Regional Water Networks: The Catalonia Case Study

C.C. Sun, V.¸ Puig, G. Cembrano., *Temporal Multi-level Coordination Techniques Oriented to Regional Water Networks: Application to the Catalonia Case Study. IWA Jounal of Hydroinformatics (submitted)*. 2013

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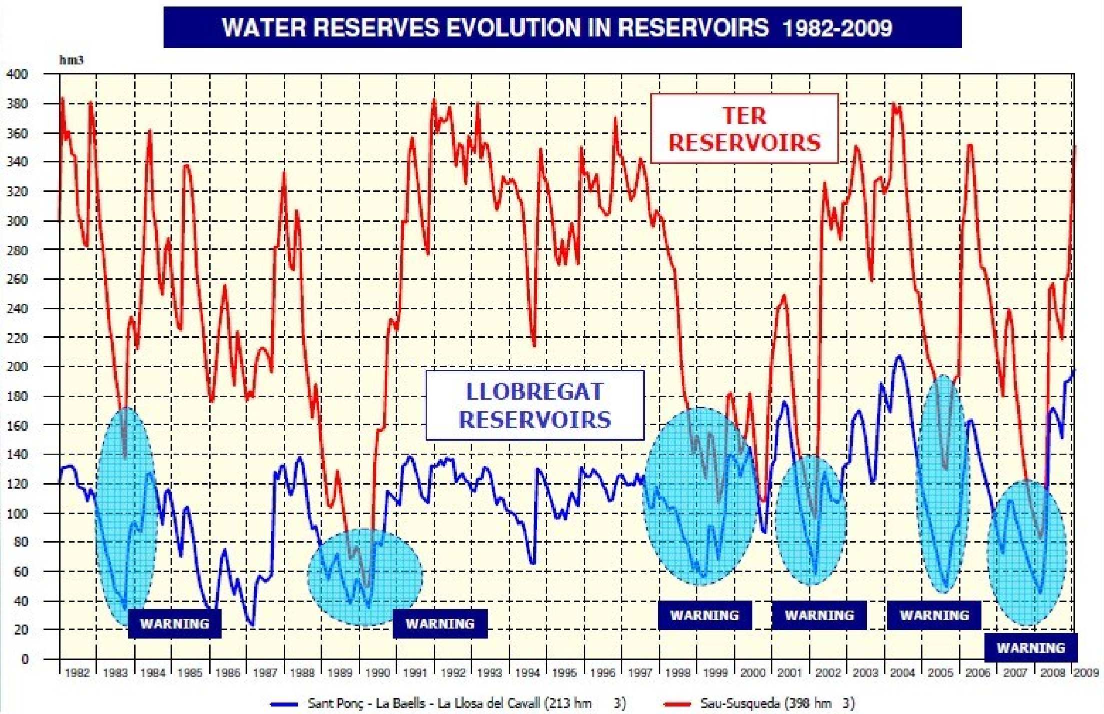
UPC

Motivation

Main supply systems

Motivation

Chronic water shortages are periodically affecting 4.5 million of people in Catalonia.



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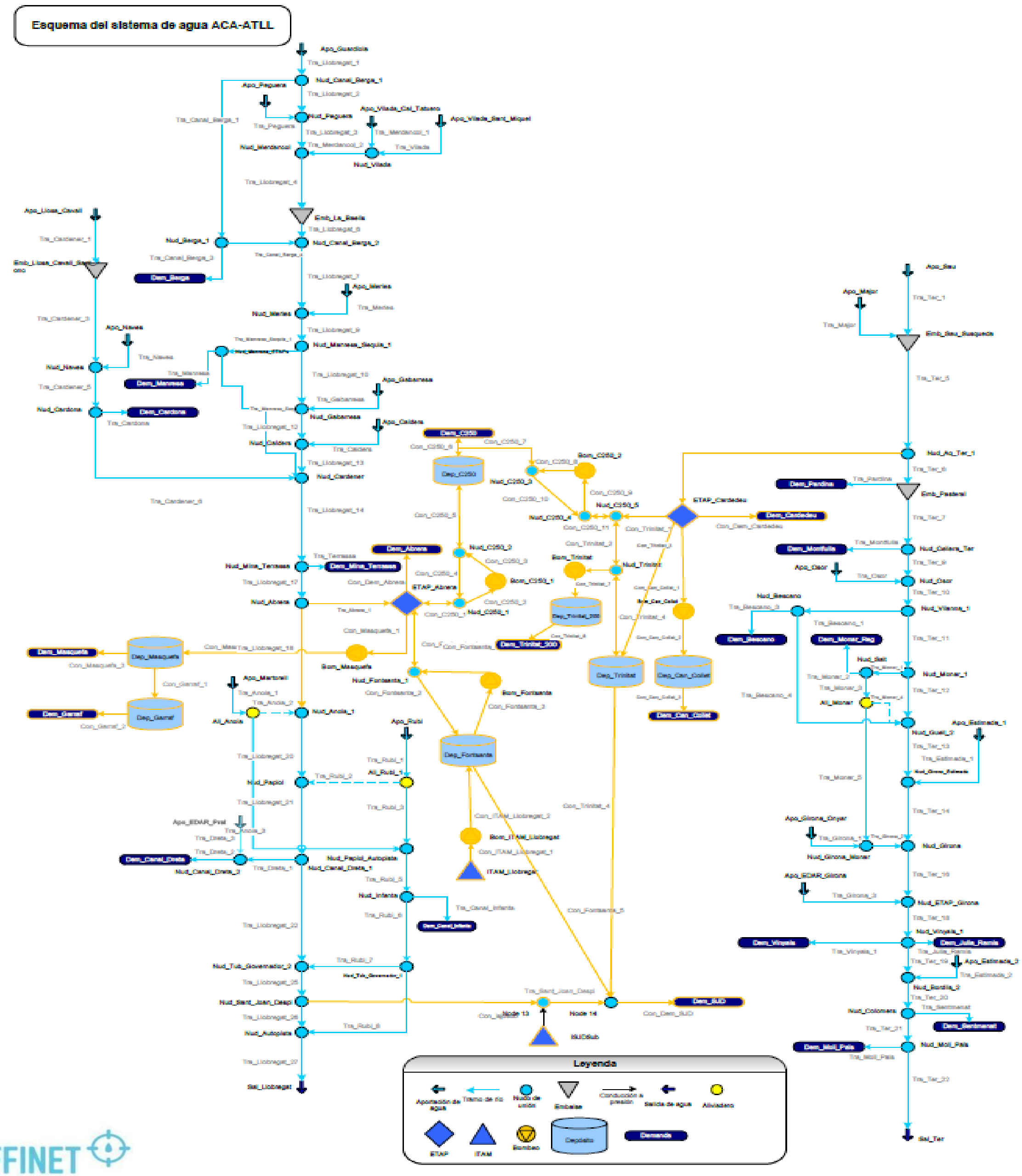
Motivation

- The authorities were considering building a desalination plant or construction of a pipeline to divert water from the Rhone in France to Barcelona
- Finally, authorities built a desalination plant.

Motivation



1. Supply

upper layer, composed by water sources, large reservoirs and also natural aquifers, rivers, wells, etc.

2. Production/transportation

middle layer, links the water treatment and desalinization plants with the reservoirs distributed all over the city.

3. Distribution

lower layer, used to meet demands of consumers.

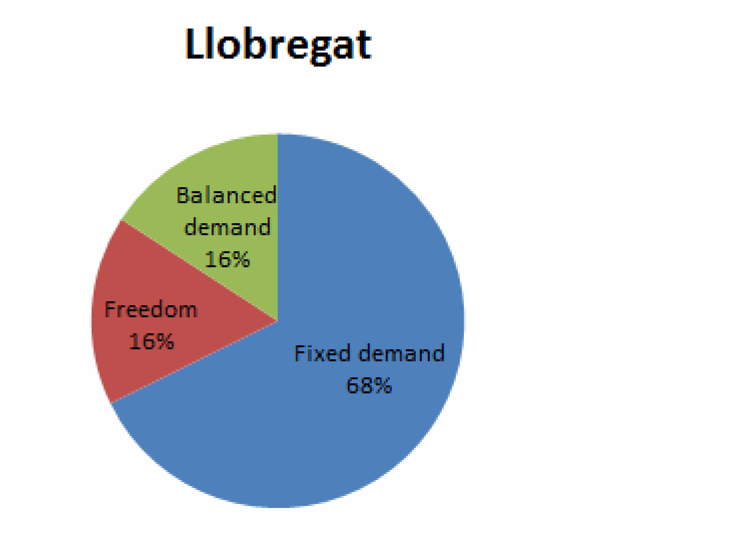
Control Objectives (1)

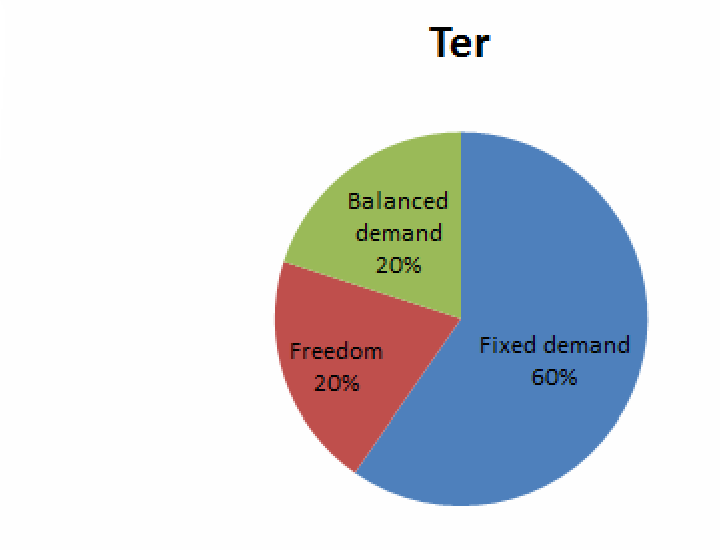
- *Operational safety* (J_{safety}): This criterion refers to maintain appropriate water storage levels in dams and reservoirs for emergency-handling. Operated in both supply and transportation layers.

- *Demand management* (J_{demand}): This is especially important in the supply layer when urban and irrigation demands exist since urban demands must be fully satisfied while irrigation demands allow some degree of slackness.

Control Objectives (2)

- *Balance management* ($J_{balance}$): This is operated only at supply layer which is necessary for keeping rivers or reservoirs consumed in a balanced way and escaping water deficit problem for both of the two rivers in a longer time.





Control Objectives (3)

- *Minimizing waste* (J_{mwaste}): Take into account that the river water eventually goes to the sea, this term gets to avoid unnecessary water release from reservoirs (release water that does not meet any demand and is eventually wasted).
- *Environment conservation* ($J_{ecological}$): Water sources such as boreholes, reservoirs and rivers are usually subject to operational constraints to maintain water levels and ecological flows.

MPC Multi-objective Function

Optimization function:

$$
\begin{aligned}
J &= J_{safety} + J_{demand} + J_{mwaste} + J_{balance} \\
&= \varepsilon_{\widetilde{x}}(k)^\top W_{\widetilde{x}} \varepsilon_{\widetilde{x}}(k) + \varepsilon(k)^\top W_f \varepsilon(k) \\
&+ (\widetilde{u}_{i \ldots j}(k) - \widetilde{u}_s(k))^\top W_{\widetilde{w}} (\widetilde{u}_{i \ldots j}(k) - \widetilde{u}_s(k)) \\
&+ \left(\begin{pmatrix} 0 & \ldots \frac{1}{xi'_{max}} & \ldots \frac{-1}{xj'_{max}} & \ldots & 0 \end{pmatrix} \widetilde{x}(k)\right)^\top w_{\widetilde{m}} \\
&\times \left(\begin{pmatrix} 0 & \ldots \frac{1}{xi'_{max}} & \ldots \frac{-1}{xj'_{max}} & \ldots & 0 \end{pmatrix} \widetilde{x}(k)\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\varepsilon_{\widetilde{x}}(k) &= \widetilde{x}(k) - \widetilde{x}_r \\
\widetilde{u} &= \Theta \Delta \widetilde{u} + \Pi \widetilde{u}(k-1) \\
\Delta \widetilde{u}(k) &= \widetilde{u}(k) - \widetilde{u}(k-1)
\end{aligned}
$$

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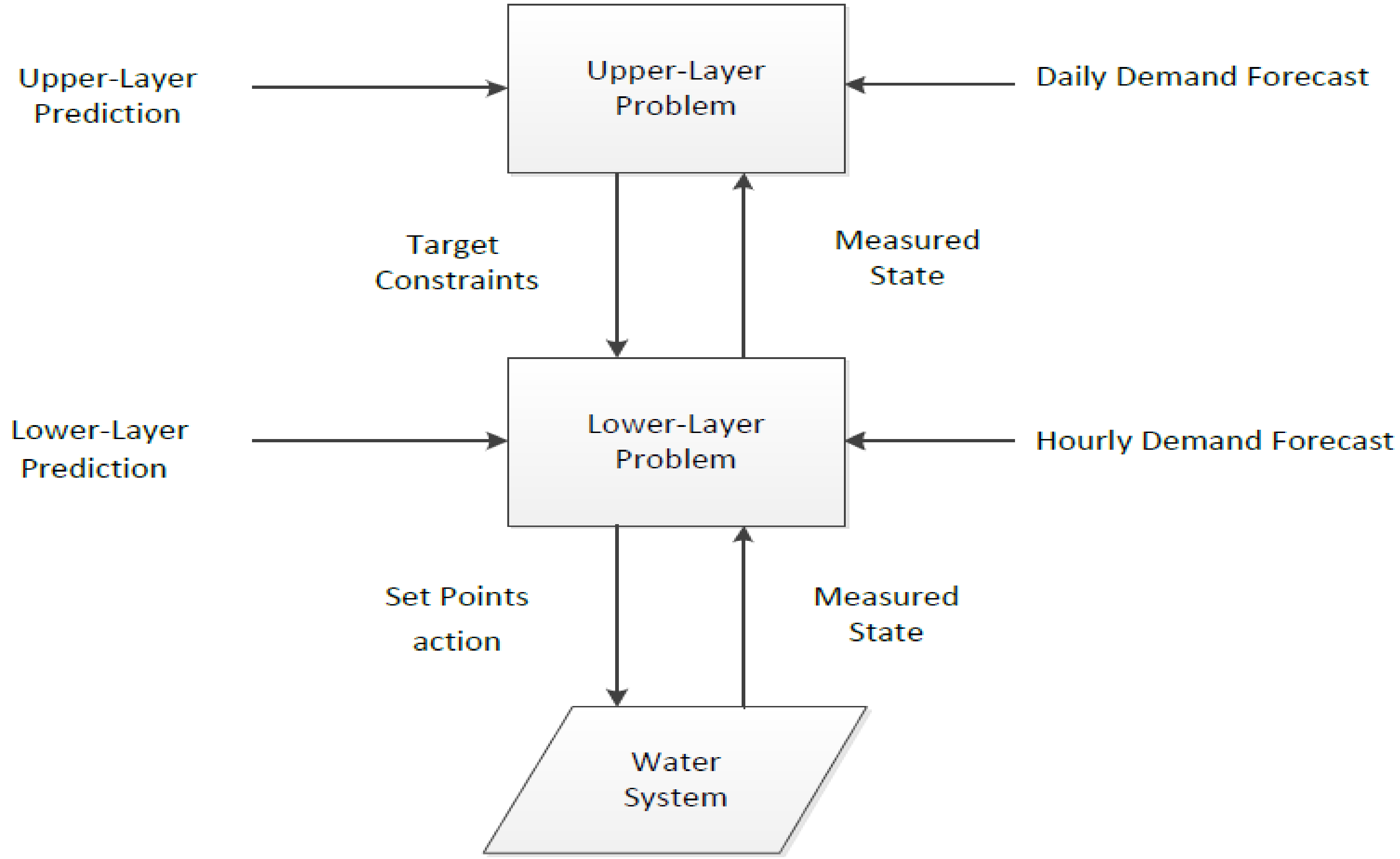
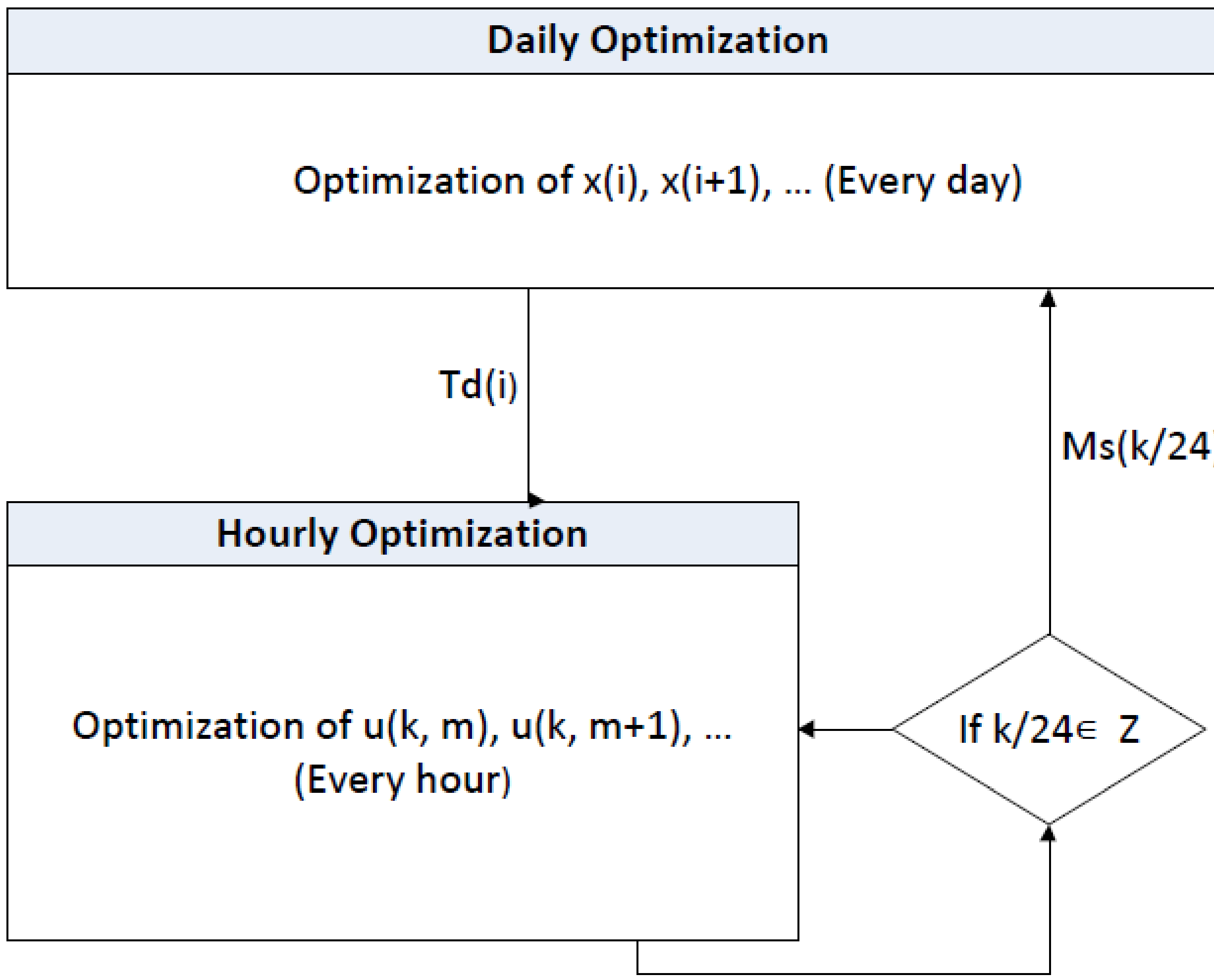
Preliminary Results

Coordination Strategy



Preliminary Results

Coordination Strategy



Preliminary Results

Balance management:

Table 1: Balancing comparison (all values in e.u.)

Sc.	With Balancing Management						
Es.	Source	Fixed Demand	Variable Demand	BD	PR	PB	SA
L.	3008	2981	724	697	58.93%	53.48%	242 Days
T.	3532	3518	1196	1182			
Sc.	**Without Balancing Management**						
Es.	Source	Fixed Demand	Variable Demand	BD	PR	PB	SA
L.	3008	2981	7.6	-19.4	-1.02%	53.48%	177 Days
T.	3532	3518	1914	1900			

Preliminary Results

Performance comparation:

Table 2: Closed-loop performance results (all values in e.u.)

Dem.	Curr.			MPC with Coordination			MPC without Coordination		
Epis.	Wat.	Ele.	Tot.	Wat.	Ele.	Tot.	Wat.	Ele.	Tot.
11/08/02	240	100	340	213	44	257	141	40	181
11/08/03	239	106	345	237	47	284	170	39	209
11/08/04	246	94	340	238	48	286	171	41	212
11/08/05	264	110	374	253	66	319	168	42	210
Mean Imp.				-5%	-50%	-18%	-34%	-61%	-42%

Embedding Fault tolerance in the MPC of Water Networks

D. Robles, V. Puig, C. Ocampo-Martinez, L.E. Garza Actuator Fault Tolerance Evaluation Methodology for Overactuated Systems using Linear Constrained Model Predictive Control, Control Engineering Pracitce (under revision). 2013

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UPC

Fault-tolerance in MPC

- Fault-tolerance against faults can be embedded in MPC it relatively easy (Maciejowski, 2002).

- This can be done in two ways:

(1) Redefining the constraints to represent certain kinds of faults, being this particularly appropriate for actuator fault.

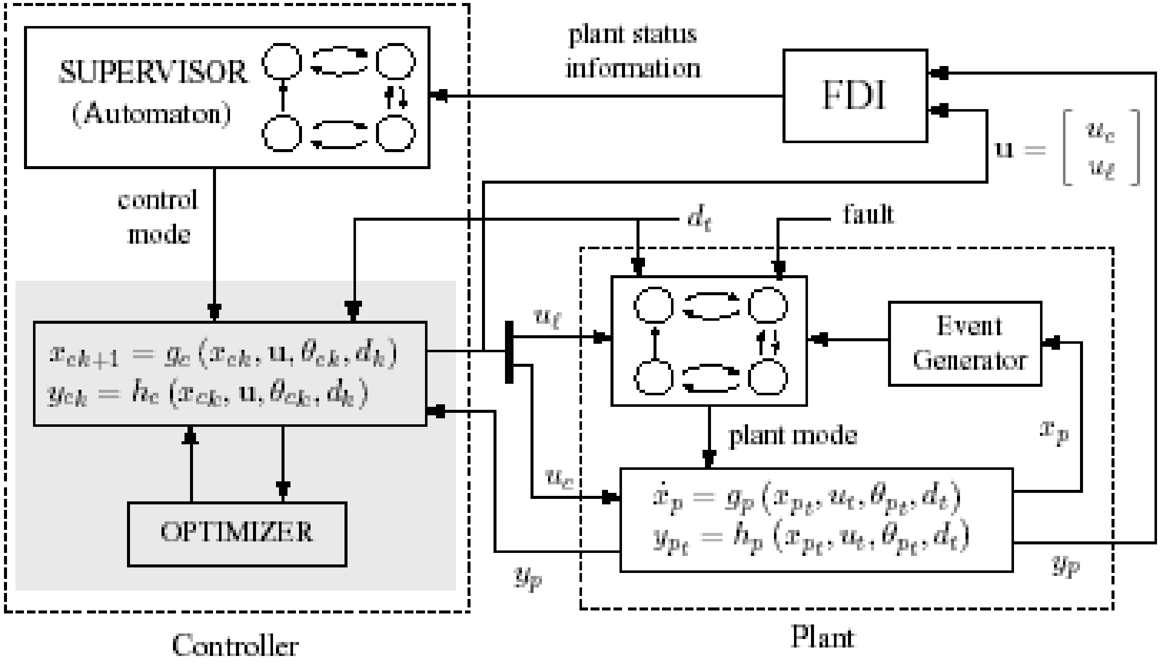
For example, in the case that a actuator is stuck at a given position, it can be represented in the optimization program by changing:

- the lower and upper constraints,
- or if the value at which the actuator is stuck is known, inserting it as both a lower an upper constraint;

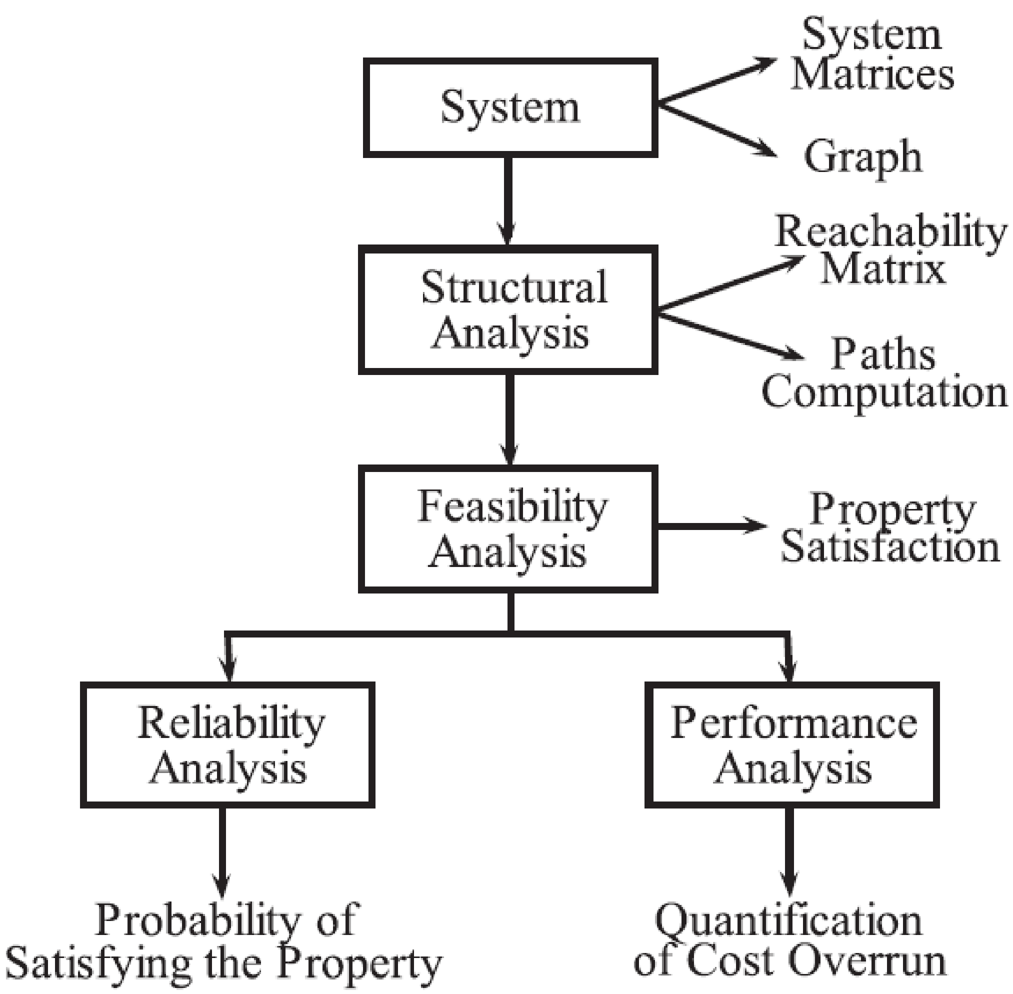
(2) Changing the control objectives to reflect limitations because of the faulty conditions.

Embedding Fault-tolerant MPC in the Hybrid Framework

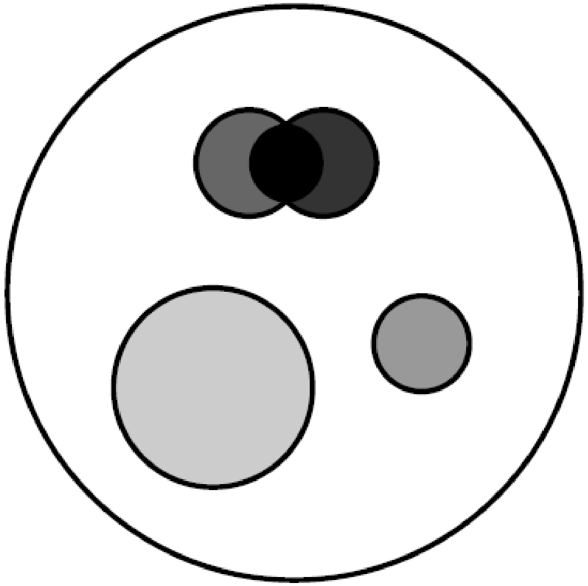
- After fault modes has been incorporated in the model used by the controller, an Active Fault Tolerant HMPC (AFTMPC) architecture is proposed to handle faults.
- The control system should incorporate an FDI module that will be used to as an external event generator to change from fault modes



Fault Tolerance Evaluation Methodology



Identifying Critical and Redundant Elements



- Redundant Elements

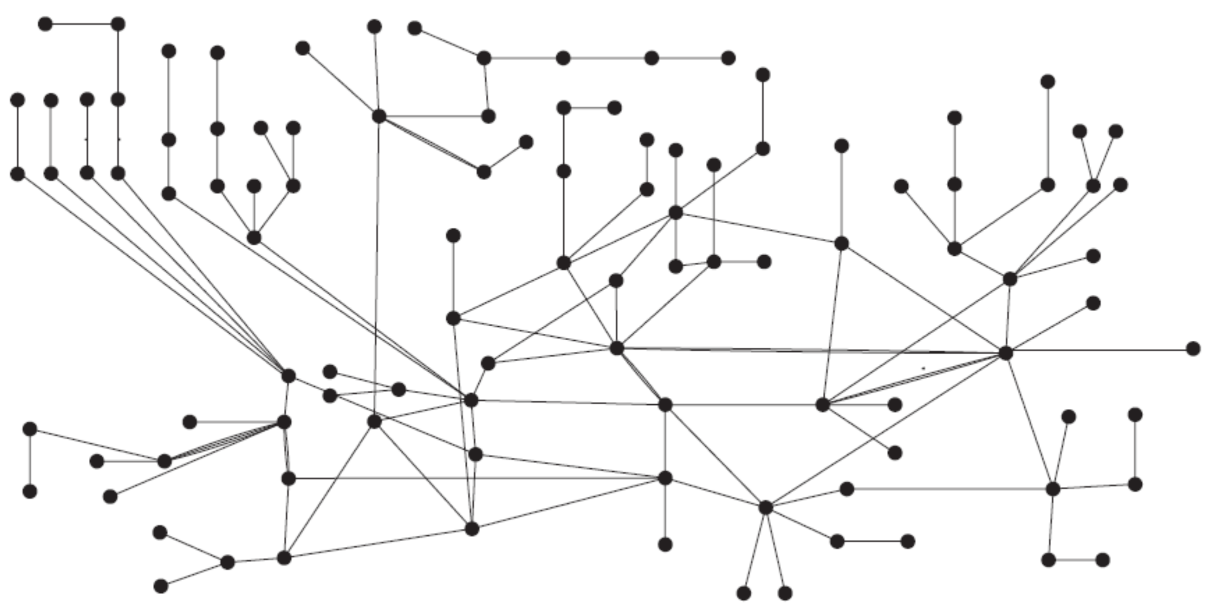
Critical Elements through:

- Reachability Analysis
- Feasibility Analysis
- Performance Analysis
- Reliability Analysis
- Performance and Reliability Analysis

Structural Analysis

Algorithm 1 Reachability Analysis using Structural Approach

Obtain the digraph $G = (\mathscr{X} \cup \mathscr{U}, \mathscr{E})$ of the system model used for the MPC for a given AFC
From the system digraph $G = (\mathscr{X} \cup \mathscr{U}, \mathscr{E})$, find the reachability matrix R
for each $x_i \in \mathbb{R}^n, i = 1, \ldots, n_x$ **do**
    if $\nexists u_j \in \mathbb{R}^m, j = 1, \ldots, n_u \mid r_{ij} = 1$ **then**
        AFC is *non reachable*
    end if
end for



Tolerance Evaluation (1)

- The objective is to assess the tolerance of a certain actuator fault configuration considering a non-linear predictive/optimal control law with constraints.
- This problem has been already treated in the literature for the case of LQR problem without constraints (Staroswiecki,2003), thanks to the existence of analytical solution.
- However, Model Predictive Control (MPC) problem does not have, in general, an analytical solution, which makes difficult to do this type of analysis
- Nonlinearity and constraints (on states and control signals) are always present in real industrial control problems.
- The method proposed is not of analytical but of computational nature.

Tolerance Evaluation (2)

- It follows the idea based on the calculation of the control law for a predictive/optimal controller with constraints can be divided in two steps:
  - first, the calculation of solutions set that satisfies the constraints (feasible solutions) and
  - second, the optimal solution determination.

- Faults in actuators will cause changes in the set of feasible solutions since constraints on the control signals have varied.

- This causes that the set of admissible solutions for the control objective could be empty.

- Therefore, the admissibility of the control law facing the actuator faults can be determined knowing the feasible solutions set.

Constraints Satisfaction Problem

- ***Constraints satisfaction problem***:

  "*A **constraints satisfaction problem** (CSP) on sets can be formulated as a 3-tuple H = (V,D,C) where:*

  - $V = \{ v_1 , \cdots ,v_n \}$ *is a finite set of variables,*
  - $D = \{D_1 , \cdots ,D_n \}$ *is the set of their domains represented by closed sets*
  - $C =\{c_1 , \cdots ,c_n \}$ *is a finite set of constraints relating variables of V "*

- A point solution of H is a n-tuple $(v_1 ,\cdots ,v_n)$ 2 D such that all constraints C are satisfied.
- The set of all point solutions of H is denoted by S(H). This set is called the global solution set.
- The variable v_i 2 V_i is consistent in H if and only if:

$$\forall v_i \in \mathcal{V}_i \;\; \exists \; (\tilde{v}_1 \in \mathcal{D}_1, \cdots, \tilde{v}_n \in \mathcal{D}_n) \, | (\tilde{v}_1, \cdots, \tilde{v}_n) \in \mathcal{S}(\mathcal{H})$$

with *i=1...n*

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Feasibility Evaluation using Constraints Satisfaction

Algorithm 2 Feasibility Analysis

for $k = 1$ to N **do**
  $\mathscr{U}_{k-1} \Leftarrow \mathscr{U}$
  $\mathscr{X}_k \Leftarrow \mathscr{X}$
end for
$\mathscr{W} \Leftarrow \{\overbrace{x_1, x_2, \cdots, x_N}^{\tilde{x}}, \overbrace{u_1, u_2, \cdots, u_{N-1}}^{\tilde{u}}\}$
$\mathscr{D} \Leftarrow \{\mathscr{X}_1, \mathscr{X}_2, \cdots, \mathscr{X}_N, \mathscr{U}_1, \mathscr{U}_2, \cdots, \mathscr{U}_{N-1}\}$
$\mathscr{Z} \Leftarrow \left\{ (x_{k+1} = Ax_k + Bu_k)_0^{N-1} \right\}$
$\mathscr{H}_{\mathscr{A}} = (\mathscr{W}, \mathscr{D}, \mathscr{Z})$
Check the existence of a solution for CSP $\mathscr{H}_{\mathscr{A}}$ by proving the feasibility of the optimization problem (16)
if the optimization problem (16) is not feasible **then**
  AFC is *non-admissible*
else
  AFC is *admissible*
end if

Performance Evaluation using Constraints Satisfaction

Algorithm 3 Performance Analysis

for $k = 1$ to N **do**

  $\mathscr{U}_{k-1} \Leftarrow \mathscr{U}$

  $\mathscr{X}_k \Leftarrow \mathscr{X}$

end for

$\mathscr{W} \Leftarrow \{\overbrace{x_1, x_2, \cdots, x_N}^{\tilde{x}}, \overbrace{u_1, u_2, \cdots, u_{N-1}}^{\tilde{u}}\}$

$\mathscr{D} \Leftarrow \{\mathscr{X}_1, \mathscr{X}_2, \cdots, \mathscr{X}_N, \mathscr{U}_1, \mathscr{U}_2, \cdots, \mathscr{U}_{N-1}\}$

$\mathscr{Z} \Leftarrow \left\{ (x_{k+1} = Ax_k + Bu_k)_0^{N-1}, \phi(x_N) + \sum_{i=0}^{N-1} \Phi(x_i, u_i) \leq J_f \right\}$

$\mathscr{H}_{\mathscr{A}} = (\mathscr{W}, \mathscr{D}, \mathscr{Z})$

Check the existence of a solution for CSP $\mathscr{H}_{\mathscr{A}}$ by proving the feasibility of the optimization problem (16) including the constraint (17)

if the optimization problem (16) including the constraint (17) is not feasible **then**

  AFC is *non-admissible*

else

  AFC is *admissible*

end if

Reliability Analysis

- r serial subsystems defined as

$$R_g^p(T_d) = \prod_{i=1}^{r}(T_d),$$

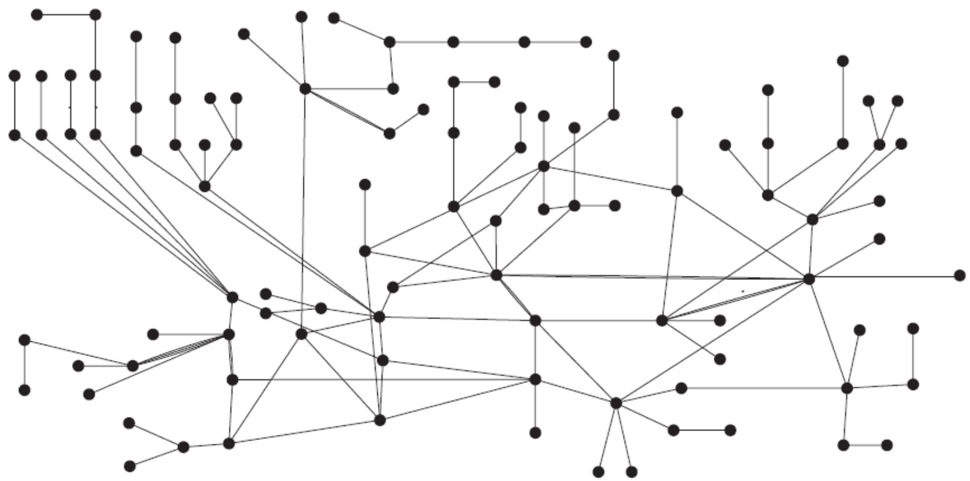
- r parallel subsystems defined as

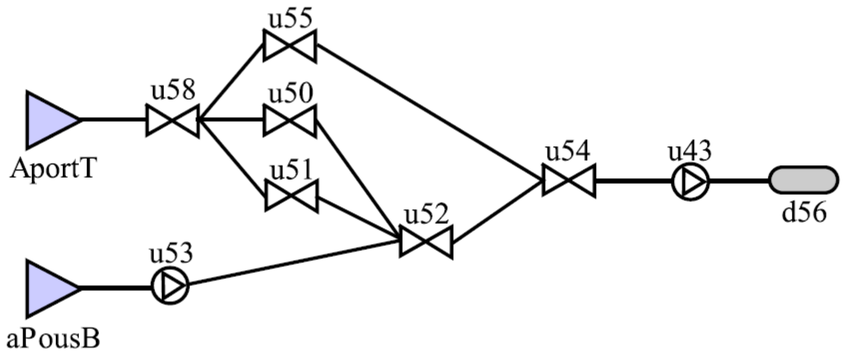
$$R_g^p(T_d) = 1 - \prod_{i=1}^{r}(1 - R_i^p(T_d)).$$

Algorithm 4 Reliability Analysis

```
for i = 1 to n_u do
    calculate (R_i^p) using (18).
end for
for g = 1 to r do
    calculate (R_g^p) using (19) and (20).
end for
if R_g^p > R_th then
    AFC is non-admissible
else
    AFC is admissible
end if
```

Tolerance Evaluation: Structural Analysis





Path 1: $AportT \to u58 \to u50 \to u52 \to u54 \to u43 \to d56$
Path 2: $AportT \to u58 \to u51 \to u52 \to u54 \to u43 \to d56$
Path 3: $AportT \to u58 \to u55 \to u54 \to u43 \to d56$
Path 4: $aPousB \to u53 \to u52 \to u54 \to u43 \to d56$

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Tolerance Evaluation: Structural Analysis

Table 1. Structural Critical Actuators (towards tanks

No.	Name	No.	Name	No.	Name	No.	Name
122	iAltures	15	iCanGuey2	62	iGuinardera1	30	iPapiol1
10	iBegues1	14	iCanGuey3	60	iGuinardera2	88	iSJD10
6	iBegues2	21	iCanRoig	101	iLaSentiu	7	iStBoi
2	iBegues3	57	iCanRuti	34	iMasGuimbau1	9	iStCliment1
1	iBegues4	37	iCarmel	31	iMasGuimbau2	5	iStCliment2
32	iBellsoleig	43	iCerdMontflorit	100	iMasJove	40	iStGenis1
61	iBonavista	42	iCerdUAB	68	iMntjcStaAmalia	38	iStGenis2
20	iCanGuell1	12	iCesalpina1	69	iMntjcTresPins	13	iStaClmCervello
17	iCanGuell2d3	11	iCesalpina2	3	iOrioles	45	iStaMaMontcada
16	iCanGuell2d5	82	iCornella100	23	iPalleja1	35	iTibidabo
18	iCanGuey1d2	39	iFlorMaig	24	iPalleja2	56	iTorreBaro1
19	iCanGuey1d5	109	iFnestrelles300	27	vPalleja70	65	iTorreoCastell
44	iVallensana1	8	iViladecans1	4	iViladecans2	25	vAbrera
54	vCerdanyola90	63	vMontigala	90	vSJD	59	vTerStaColoma
104	vSJDTot	58	vTer				

Table 2. Structural Critical Actuators (towards demands

No.	Name	No.	Name	No.	Name	No.	Name
115	vPallejaATLL	116	iPalleja3	117	iMasGuimbau3	118	iVallvidrera
119	vHorta	120	iUAB	121	iVallensana2	122	iBoscVilaro
123	iTorreBaro2	124	iCerdSabadell	125	vBesosStaColoma	126	v117Montigala
127	v70CFE	128	v55BAR	129	iMontemar	130	vAltures

Tolerance Evaluation: Performance Analysis

Table 3. Entire DWN Performance Analysis for Test 5

Actuator No.	Faulty price [e.u.]	Cost overrun [%]
41	514.44	2.43
47	515.94	2.73
74	528.05	5.14
78	557.62	11.03
86	515.08	2.55
89	556.22	10.74
97	510.49	1.64
102	539.87	7.49
103	552.21	9.95

Tolerance Evaluation: Reliability Analysis

Table 4. Relation between demands satisfaction and reliability

Demand No.	Percentage of total demand[%]	Faulty Components	R_g^p in Faulty conditions[%]
69	9.1	128	0
83	4.0069	82, 88, 90, 104	0
70	3.2537	125	0
70	3.2537	58	99.33
70	3.2537	53, 50, 51	99.99
33	1.964	108	99.98
58	1.9407	52, 58	99.33
56	1.6777	43, 54	0
56	1.6777	52, 58	98.67
64	1.4941	58, 59	0

Thank you very much

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